The Shapes of Large Urban Areas in the U.S., 1950-2010: Patterns, Causes, and Consequences

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Abstract

The compactness of the shapes of 59 large urban areas in the United States from 1950 to 2010 is measured using an index of proximity based on the mean distance from the Central Business District to all areas within the urban area. Average changes in the proximity index over time are small, but individual urban areas have experienced much larger changes in shape, becoming both more and less compact. Larger urban areas tend to be somewhat less compact. Barriers to the expansion of urban areas including water, wetlands, mountains, and protected lands are associated with lower levels of proximity and compactness. Lower proximity is associated with higher urban area densities and smaller declines in densities with distance from the Central Business District.

Introduction

The shapes of urban areas can vary greatly. Some areas are reasonably regular and compact. Other areas may have very irregular boundaries or be very elongated. Such differences raise questions: How do the shapes of urban areas vary over time? What factors might affect the shapes of urban areas? And how does the shape affect patterns of development within the urban areas?

Addressing these questions requires a measure of the shape of urban areas. Many measures of the shape of areas have been proposed and numerous authors have discussed multiple measures. One of the best reviews and evaluations of different alternatives is by Maceachren (1985). A different, useful perspective has been provided by Angel, Parent, and Civco (2010), relating shape measures to various properties of a circle, seen as the most compact shape, and arguing that the selection of a measure should be based on the property appropriate to the purpose for which the index is to be used.

In a previous paper (Ottensmann 2021), I discussed the general issue of the measurement of the shape of urban areas. The paper briefly reviewed literature devoted to measures of geographic shape, identified criteria for a measure of shape for urban areas, evaluated various types of shape indices, and proposed the use of a proximity index for this purpose. Values of the index were calculated for large urban areas in 2010, with the results supporting its utility as a measure of the shape of urban areas. This paper will not repeat that except for the description of the index and a brief summary of the reasons for its selection.

The next section of this paper describes how the proximity index is being used as a measure of the shape of the urban areas. This is followed by a description of the data for 59 large urban areas from 1950 to 2010 used for this analysis, including the delineation of the urban areas for which shape is being measured. The presentation of the results begins with the summary of the values of the shape index over the period and the examination of change and stability. Factors associated with the levels of the proximity index are considered next, with a focus on the effect of barriers to urban area expansion on the shapes of the areas. A final section examines the effects of shape on patterns of development within the urban areas—overall density and the patterns of density as estimated using the negative exponential model of density decline with distance from the center.

Measuring Shape: The Proximity Index

A measure of the shape of an urban area should meet 3 criteria of particular importance for such areas in addition to other criteria relevant to the evaluation of all shape indices. The index should measure the compactness of the urban area shape relative to the location of the Central Business District (CBD), which is a distinctive feature of these areas. Accessibility to that location is particularly important within urban areas. An index must work with urban area shapes having holes that are not urban, highly convoluted boundaries, and discontinuous parts. And any measure should work well with urban areas commonly defined in different ways, including aggregations of small areas such as census tracts, the Urbanized Areas delineated by the census, and areas derived from satellite imagery.

Consideration of the various types of measures of geographic shape evaluated by Maceachren (1985) shows numbers fail to meet these criteria. The commonly used perimeter-area measures make no reference to the location of a center and thus fail on the CBD criterion. They also have problems with irregular boundaries. Indices based on the diameters of the smallest circumscribing circle and/or the largest inscribed circle likewise do not use a central location. They are sensitive to small changes in the boundary, and holes can create problems. A measure using variation in distances from a center to the periphery can reflect CBD location. But it is sensitive to the selection of the directions to the periphery for which distance is measured, which can be a major issue for some types of urban areas. Such a measure also cannot deal well with holes, discontinuous areas, and some indentations along the boundary.

Measures of urban area shape based on the portion of the urban area within a circle having the same areas as the urban area are better. The center of the circle can be located at the CBD. Holes and discontinuous areas present no problems. These are satisfactory but crude measures of urban area shape. The limitation is that they are based only on whether the distances of parts of the shape to the CBD are less than or greater than the radius of that circle. Especially for areas outside the circle, the measures do not take into account how far the areas are from the CBD and the circle, whether just outside or much more distant.

The index chosen for this research is based on the distances from all parts of the urban area to a center, which can be the CBD. The proximity index described by Angel, Parent, and Civco (2010) uses the mean distance from all areas to that center. They suggest that it can be the CBD when used to measure the shape of urban areas and say that proximity would be "a natural measure of accessibility of a metropolitan area to its Central Business District (CBD)." Of course the mean distance to the CBD will tend to be greater for larger urban areas. To standardize for the size of the area, the proximity index also uses the mean distance from all points in a circle with the same area as the area of the urban area to the center of that circle. The index is then the ratio of the mean distance for the circle to the mean distance from the CBD for the urban area, so higher values represent areas that are more compact.

For an urban area composed of small areas such as census tracts, the areas being considered here, the mean distance $\bar{S}$ from the CBD will be the average of the distances from the tract centroids to the CBD weighted by the area of the tract:

$$\bar{S} = \frac{\sum_i s_i a_i}{\sum_i a_i} = \frac{\sum_i s_i a_i}{A}$$

where s_i is the distance from the center of tract i to the CBD, a_i is the area of tract i, and A is the total area of the urban area, the sum of the tract areas.

The mean distance to the points in a circle with the same area A as the urban area is 2/3 the radius of that circle, which is equal to the square root of the area divided by π. The proximity index P is then the ratio of this mean distance for the circle to the mean distance for the urban area:

$$P = \frac{2/3\sqrt{A/\pi}}{\bar{S}}$$

The value of P ranges from 0 to 1, with the value of 1 for a circular area, maximum compactness, and 0 for the least compact area. Note that the inverse of P is a measure of how much longer the mean distance to the CBD for the urban area is than the mean distance if that area were circular.

For those urban areas having multiple CBDs, the overall proximity index for the urban area is the average of the values of the index calculated for the areas of the tracts associated with each of those CBDs weighted by the area of those subareas. Let P_j be the proximity index for the area of the tracts associated with CBD j and let A_j be the area of those tracts. Then the proximity index P for the entire urban area is:

$$P = \frac{\sum_j P_j A_j}{\sum_j A_j}$$

Since the values of the proximity indices for the subareas vary from 0 to 1, the value for the overall proximity index will have the same range, with a value of 1 only when each of the subareas is circular (very unlikely).

Data and Context

This study is part of a larger research project that looks at patterns of development within 59 large urban areas in the United States from 1950 to 2010. These areas, the urban areas delineated for each census year, are the areas for which the proximity index is calculated and provide the context within which the analysis is being undertaken. This section starts by identifying the urban areas included in the dataset, describes the housing unit data for census tracts that is the core of the data, and discusses how these data have been used to define urban areas from 1950 to 2010. The identification of the locations of the Central Business Districts (CBDs) is then addressed.

This research uses a dataset for the analysis of urban patterns over time that was developed with data on numbers of housing units in census tracts for large urban areas in the United States from 1950 to 2010. The tracts for urban portions of metropolitan areas were identified within the Combined Statistical Areas (CSAs) as delineated by the Office of Management and Budget for 2013 (U.S. Bureau of the Census 2013). CSAs were used rather than the more commonly employed Metropolitan Statistical Areas (MSAs) as it was felt they better represented the full extent of the metropolitan areas, including those instances in which 2 or more MSAs should more properly be considered to be parts of a single area (Ottensmann 2017). For those MSAs which were not incorporated into a CSA, the MSA was used.

The 59 CSAs and MSAs with 2010 populations over one million were selected for the creation of the dataset. A number of these areas had multiple large centers associated with separate urban areas that had grown together. This posed the issue of identifying those cases in which a second or third urban area could be considered sufficiently large in relation to the largest area to be included as an additional center around which urban development occurred. The decision was made by comparing the populations of census Urbanized Areas (either from the current census or the last census in which the areas were separate) with the largest area. An area was considered to be an additional center if its population were greater than 28 percent of the population of the largest area. The three areas included with the lowest percentages were Akron (with Cleveland), Tacoma (with Seattle), and Providence (with Boston). Areas with multiple centers have each center included in the name of the urban area.

The primary data source for this research was the Neighborhood Change Database developed by the Urban Institute and Geolytics (2003). This unique dataset provides census tract data from the 1970 through 2000 censuses, with the data for 1970 through 1990 normalized to the 2000 census tract boundaries. Population and housing unit data from the 2010 census were added by aggregating the counts from the 2010 census block data (U.S. Bureau of the Census 2012).

Housing units and housing unit densities—the numbers of housing units divided by the land areas of the tracts in square miles—are used in this research rather than the more commonly employed population and population density measures for two reasons. Housing units better represent the physical pattern of urban development as they are relatively fixed, while the population of an area can change without any changes in the stock of housing. Other studies of urban patterns have made similar arguments for choosing housing units over population, for example Galster, *et al.* (2001), Theobald (2001), Radeloff, Hammer, and Stewart (2005), and Paulsen (2014).

Using housing units also allows the extension of the analysis to census years prior to 1970. The census provides data on housing units classified by the year in which the structure was built, and these data are included in the Neighborhood Change Database. The 1970 year-built data can be used to estimate the numbers of housing units present in the census tracts for 1940, 1950, and 1960. Several prior studies have used the housing units by year-built data to make estimates for prior years in this manner, though they have used more recent census data to make the estimates, not the earlier 1970 census data (Radeloff, *et al.* 2001; Theobald 2001; Hammer, *et al.* 2004; Radeloff, Hammer, and Stewart 2005).

Sources of error in these housing unit estimates for earlier years from the year-built data arise from imperfect knowledge of the year in which the structure was built and from changes to the housing stock due to demolitions, subdivisions, and conversions to or from nonresidential uses. These errors increase for estimates farther back in time. Numbers of housing units for 1970 to 1990 were estimated from the 2000 year-built data and compared with the census counts in the Neighborhood Change Database. The judgment was made that estimates 2 decades back involved acceptable levels of error, but this was not the case for 3 decades back. As a result, the decision was made to use the housing unit estimates for 1950 and 1960 but not for 1940.

Urban areas have been defined for the urban patterns research for each census year since 1950 consisting of those tracts contiguous to an urban center meeting a minimum housing unit density threshold. (This is comparable to the way in which the census defines Urbanized Areas using blocks and larger units and Paulsen (2012) defined urban areas using block groups.) For the definition of Urbanized Areas for the 2000 and 2010 censuses, a minimum population density of 500 persons per square mile was required for an area to be included (U.S. Bureau of the Census 2002, 2011). Using the ratio of population to housing units for the nation as a whole in both 2000 and 2010 of 2.34 persons per unit, a density of 500 persons per square mile is almost exactly equivalent to 1 housing unit per 3 acres or 213.33 units per square mile. This was used as the minimum urban density threshold. Note that this is a measure of gross density, not lot size, as the areas of roads, nonresidential uses, and vacant land are included.

To provide for a set of urban areas that represents the cumulative expansion of the urban areas over time, a further condition was imposed that if a census tract did not exceed the minimum housing unit density and had not been included in the urban area in any given year, it would not be included in urban areas delineated in earlier years even if the density exceeded the minimum. The rule has been imposed in this direction —if rural, then not urban earlier—rather than in the opposite direction—if urban, then urban later—because the more recent data are considered to be generally more accurate.[1]

The location of the CBD must be specified to measure distances. One of the only efforts by the Census to do so came in a report for the 1983 economic censuses (U.S. Bureau of the Census 1983). This lists the census tracts comprising the CBD for many larger cities. This information was used to identify the CBD tracts for those urban areas included and for which the tract numbering and boundaries were the same for 2000. For the other urban centers, the tract or tracts for the CBD were identified by determining the location of the city hall or other major government buildings and examining the pattern of major roads, which generally converge on the CBD. The centroid of the CBD tract or tracts was taken as the center. Distances to the center were measured in miles from the centroids of each of the census tracts in the urban area.

For the 16 areas with 2 or 3 centers, the calculation of the distances from the census tracts to the CBD and the values of the proximity index required the partitioning of those urban areas. Each tract had to be assigned to one of the multiple CBDs. For those areas for which separate Urbanized Areas were delineated for the census in 2010, the tracts were assigned using the Urbanized Area boundaries. For the urban areas with a single Urbanized Area encompassing the entire urban area, tracts were assigned to the nearest center. In a few instances, modifications were made based on the geography of

[1] More detail on the construction of the dataset and the delineation of the urban areas is provided in Ottensmann (2014).

the area. For example, for Tampa-St. Petersburg, tracts on either side of Tampa Bay were assigned to the center on that side, even though in a few instances they were actually closer to the other center.

Urban Area Shape from 1950 to 2010

This begins describing the pattern of urban shape for the 59 large urban areas in the United States from 1950 to 2010, the values of the index of proximity. Table 1 presents summary statistics for the index for each census year. On average, not that much has changed over the 60 years. Perhaps a slight decline in both the mean and the median, but those changes were not great. And the same was true for most of the urban areas, with the first and third quartiles and the maximum showing much the same pattern. The minimum proximity did increase by a somewhat larger amount from 1950 to 2010, but the pattern was irregular and it is hard to see this as a trend.

Table 1. Summary Statistics for Proximity Index.

Year	Mean	Minimum	First Quartile	Median	Third Quartile	Maximum
1950	0.78	0.45	0.70	0.80	0.86	0.95
1960	0.78	0.50	0.71	0.79	0.85	0.96
1970	0.79	0.54	0.72	0.80	0.87	0.97
1980	0.77	0.48	0.72	0.78	0.85	0.93
1990	0.76	0.47	0.69	0.76	0.83	0.91
2000	0.76	0.53	0.69	0.77	0.83	0.93
2010	0.76	0.53	0.69	0.77	0.83	0.93

The next question becomes how the proximity indices for the individual urban areas changed over time. Table 2 gives the summary statistics for the change in the index from decade to decade and over the entire period from 1950 to 2010. This gets more interesting. The average changes as measured by the mean and median were virtually zero, consistent with the fairly steady average values for the index shown in the preceding table. And this was true for the middle half of the distribution as well. The first and third quartiles for the change from one decade to the next were also very small. But this was hardly the case for the urban areas at the extremes. Proximity index values decreased or increased from decade to decade by amounts as high as 0.18, with the maximum absolute change from 1950 to 2010 of 0.21. Only 3 decades saw increases of less than 0.10. So while the average proximity remained fairly stable, individual urban areas experienced significant changes in both directions.

It is important to understand how major changes in the shape proximity index for an urban area can change from one decade to the next. The addition of census tracts that previously had not met the minimum density criterion to an urban area are likely to have relatively moderate effects on its shape and proximity value. However, as urban areas expand, other smaller urban areas that were previously separate can now become contiguous and be added to the urban area. And some of these areas can be quite large, even separate Urbanized Areas with populations exceeding 50,000. The additions of these larger areas can have far more significant effects on the shape of an urban area.

Table 2. Summary Statistics for Change in Proximity Index.

Decade	Mean	Minimum	First Quartile	Median	Third Quartile	Maximum
1950-1960	-0.00	-0.15	-0.03	-0.00	0.03	0.15
1960-1970	0.01	-0.09	-0.02	0.00	0.03	0.16
1970-1980	-0.01	-0.17	-0.04	-0.01	0.02	0.07
1980-1990	-0.01	-0.18	-0.02	-0.00	0.01	0.05
1990-2000	-0.00	-0.12	-0.02	0.00	0.02	0.05
2000-2010	0.00	-0.13	-0.01	0.00	0.02	0.14
1950-2010	-0.02	-0.21	-0.09	-0.02	0.03	0.20

Table 3. Correlations of Proximity Index Values in Different Years.

Proximity Indices	Correlation	
1950 with 1960	0.86	***
1960 with 1970	0.93	***
1970 with 1980	0.92	***
1980 with 1990	0.92	***
1990 with 2000	0.94	***
2000 with 2010	0.93	***
1950 with 2010	0.68	***

***Significant at the 0.001 level

This raises the question of the extent to which the shapes of urban areas as measured by the proximity index remain stable over time. Table 3 presents the correlations of the shape index values of urban areas from one census year to the next and from the beginning in 1950 to the final value for 2010. The decade-to-decade correlations were high, all but one exceeding 0.9, indicating that most urban areas had relatively consistent shape index values from one decade to the next. However, the correlation of the 1950 proximity to the 2010 proximity was much lower, 0.68, evidence of significantly larger changes in shape over longer periods of time.

Further insight is provided by looking at individual urban areas and how their proximity has changed over time. Table 4 lists the 10 urban areas with the highest urban proximity index values in 2010 and the 10 areas that were the lowest, that were the least compact. First, observations about the urban areas on these lists. The most compact urban areas with the highest proximity values tended to be in the South and West. Quite a few were somewhat smaller areas, though Las Vegas, Atlanta, and Houston were clear exceptions. But the areas that had the lowest proximity indices and were the least compact also tended to be in the South and West, with New York and Boston-Providence being obvious exceptions.

The table also provides the ranking of the areas in terms of proximity for 1950, along with the ranking for 2010 which is obviously from 1 to 10, highest or lowest proximity. Interestingly, exactly half of the urban areas had reasonably stable proximity ranks from 1950 to 2010 while the other half saw much larger changes in position over the period. Five of the 10 areas were among the top 10 areas with respect to the highest or lowest proximity in both years. But most of the others experienced significant changes in their rankings, especially among the most compact areas in 2010. Positions

Table 4. Urban Areas with Highest and Lowest Proximity Index Values in 2010, with Ranks for 1950 and 2010.

Urban Area	Proximity Index in 2010	1950 Rank	2010 Rank
Areas with Highest Proximity Index Values			
Raleigh-Durham	0.93	5	1
Rochester	0.93	9	2
Las Vegas	0.92	6	3
Columbus	0.91	29	4
Dayton	0.91	13	5
Indianapolis	0.89	17	6
Portland	0.89	31	7
Atlanta	0.87	25	8
Houston	0.87	8	9
Greensboro--Winston-Salem--High Point	0.87	2	10
Areas with Lowest Proximity Index Values			
Salt Lake City-Ogden-Provo	0.53	17	1
Miami-Fort Lauderdale-West Palm Beach	0.54	7	2
San Diego	0.54	16	3
El Paso	0.58	2	4
San Francisco-Oakland-San Jose	0.60	3	5
Sacramento	0.64	14	6
New York	0.64	18	7
Boston-Providence	0.64	20	8
Tampa-St Petersburg	0.64	4	9
Norfolk-Virginia Beach	0.64	1	10

on the lists jumped from 29 to 4, 31 to 7, and 25 to 8 for Columbus, Portland, and Atlanta. Among the least compact areas, the 5 not in the bottom 10 in 2010 dropped from being the 14th to 20th lowest in 1950. So the shape of urban areas as measured by the proximity index showed a mixture of stability and churn.

It is instructive to see what the most and least compact urban areas looks like. Figure 1 shows the outlines of several of the urban areas with the highest and lowest proximity values in 2010. They were selected as being representative of the varying shapes of the areas at each extreme. Raleigh-Durham and Columbus were the first and fourth most compact, with index values of 0.93 and 0.91. With Raleigh-Durham, remember that the index is measured as the proximity to each of the Central Business Districts for the associated census tract. So while the overall shape seems quite irregular, the 2 pieces are not as much. Columbus, on the other hand, is quite close to being a square. The Salt Lake City-Ogden-Provo urban area is the least compact with an index value of 0.53. Its extremely elongated shape is obvious, but again, the index is a weighted average of the values for the areas associated with each center, but those areas themselves were still elongated. San Diego at 0.54 was nearly as irregular and non-compact in terms of proximity.

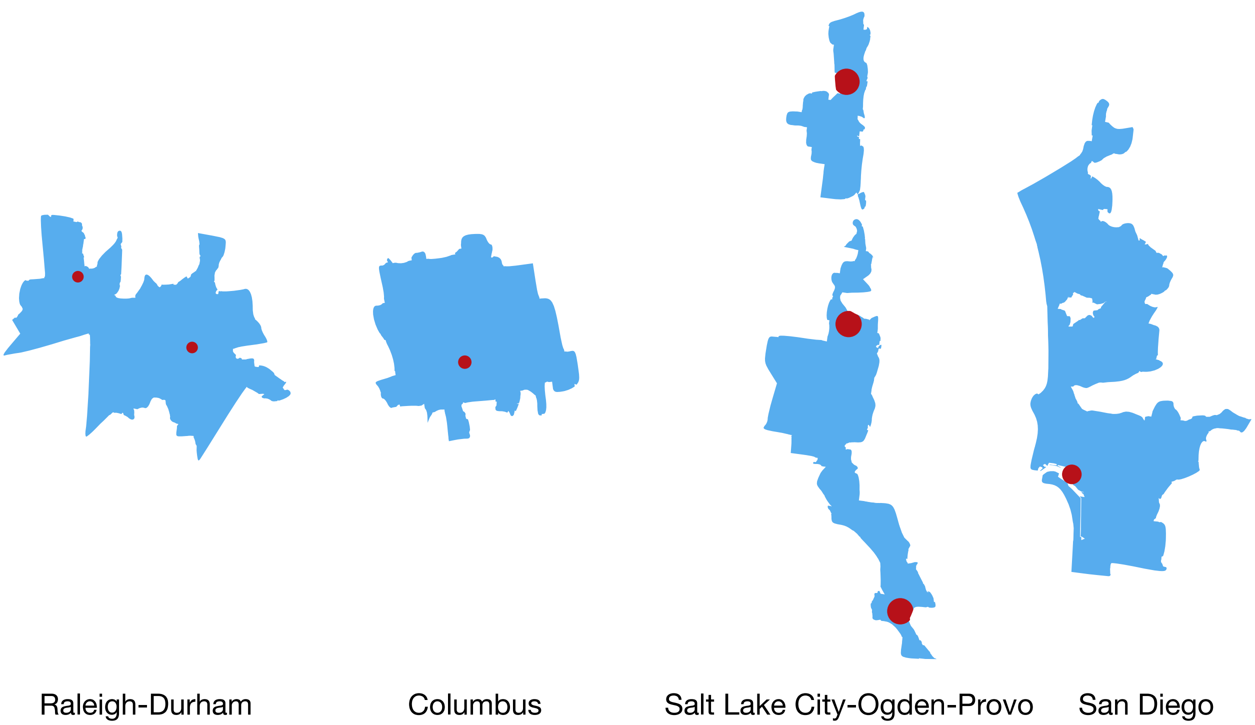

Figure 1. Examples of Urban Areas with the Among the Highest (left) and Lowest (right) Proximity Index Values.

Factors Affecting the Shape of Urban Areas

Next comes the question of why urban areas differ in their shape, the degree of proximity. Size could affect shape, with larger areas possibly having lower levels of proximity. Changes in proximity are more likely for urban areas experiencing greater increases in their areas. And barriers to urban development will influence how an urban area expands and thus its shape.

The size of the urban area could influence levels of proximity in several ways. As an urban area expands, it becomes more likely that barriers to that expansion are encountered, limiting development and decreasing the level of compactness. And a larger urban area has a longer perimeter and greater opportunity for deviation from a more compact shape. The examples of the areas with high and low levels of proximity provide mixed evidence on the effect of size, however. New York, Miami-Fort Lauderdale-West Palm Beach, Boston-Providence, and San Francisco-Oakland-San Jose, some of the largest urban areas, are on the list of the areas with the lowest proximity values. But Houston and Atlanta are among the 10 areas with the highest proximity.

Table 5 gives the correlations between the proximity index and 2 measures of the size of urban areas, the land area and the number of housing units. The correlations were all quite modest, ranging from -0.16 to -0.28. And all were negative, so proximity did seem to be somewhat lower for larger areas. None of the correlations with land area were statistically significant. For housing units, the correlations for the last 3 years from 1990 to 2010 were significant at the 0.05 level. Why the correlations were significant for housing units and not for land area is not clear, since land area might reasonably be

Table 5. Correlations of Proximity Index with Measures of the Size of Urban Areas, Land Area and Number of Housing Units.

Year	Correlation with Land Area	Correlation with Housing Units
1950	-0.16	-0.16
1960	-0.18	-0.19
1970	-0.25	-0.24
1980	-0.22	-0.24
1990	-0.21	-0.28 *
2000	-0.18	-0.26 *
2010	-0.20	-0.28 *

*Significant at the 0.05 level

expected to be more directly related to shape. The conclusion would seem to be that the proximity of an urban area is weakly related to its size in an inverse direction.

The next effect on proximity to be considered seems obvious. Urban areas that experience greater increases in their land area can be expected to see larger changes in levels of proximity. The more new land is added to an area, the greater are the possibilities for changes to the shape. In the opposite situation, an urban area that adds little new area necessarily can see little change in proximity.

In considering the relationship, it is important to recognize that this argument only speaks to the magnitude of the change in shape and the proximity index. It says nothing about the direction of the change. An urban area experiencing substantial growth could see significant increases or decreases in proximity. Therefore, Table 6 presents the correlations between the absolute value of changes in proximity to the percent change in the land area for the urban area. The correlations were positive, substantial, and statistically significant. Of the 6 correlations, 5 are 0.4 or more and were significant at least at the 0.01 level. The results are clearly as expected. Greater increases in land area are associated with larger changes, in either direction, in the proximity index.

Table 6. Correlations of Absolute Value of Change in Proximity Index with Percent Increase in Land Area.

Decade	Correlation	
1950-1960	0.40	**
1960-1970	0.44	***
1970-1980	0.30	*
1980-1990	0.59	***
1990-2000	0.53	***
2000-2010	0.57	***

*Significant at the 0.05 level

**Significant at the 0.01 level

***Significant at the 0.001 level

It is easy to see why barriers to the expansion of an urban area would result in lower levels of proximity. If an urban area is prevented from growing in one direction, demand will have to be accommodated by expansion in other directions, making the area more irregular and decreasing proximity. An obvious and common barrier would be water. An urban area located on a body of water, very likely due to the desire for a port for trade, would be prevented from growing in that direction.

The effects of 4 types of barriers on the proximity of urban areas is examined here: water, wetlands, mountains, and protected lands. Water has already been mentioned. Urban areas located in proximity to large bodies of water such as the ocean, the Great Lakes, and large estuaries are considered to have water as a barrier to urban expansion. Less common but important for a few areas are large wetlands that limit expansion. The Everglades constitute such a barrier for westward growth of the Miami-Fort Lauderdale-West Palm Beach area, for example. Mountains present barriers to urban expansion for significant numbers of urban areas, primarily in the West. Finally, some urban areas are bordered by significant amounts of land that is protected from urban development by designation as national or state parks, monuments, or forests. Protected status will at times also be areas that are wetlands or mountains, so there can be overlap.

Determination of the presence of these barriers to urban expansion was a subjective judgment. Various types of maps were examined and an area was deemed to have a particular type of barrier if the barrier was judged be of significant magnitude to significantly affect patterns of development of the urban area. Small, limited areas of wetlands, mountains, or protected lands were not identified as barriers. An appendix table lists the urban areas deemed to have each of the types of barriers.

The most direct way of examining the individual effects of the barriers on shape is to compare the mean proximity values for the urban areas with each type of barrier to the mean for the other areas without that barrier. Table 7 presents the means for proximity in 1950 and 2010 for the 4 types of barriers along with the differences in the means and the significances of the difference. As expected, the presence of water as a barrier had a major effect on the shape of the urban areas. The mean index in 2010 for the areas with water as a barrier was 0.65 while the mean for the other areas was 0.79, a difference of 0.14. The difference was similar for 1950. In both years, the differences in the means were highly statistically significant at the 0.001 level.

Wetlands exhibited similar large differences, with the mean for the areas affected by wetlands 0.14 lower in 2010 than the mean for the other areas. The difference was even slightly larger in 1950. Only the 1950 difference was statistically significant, at the 0.05 level, but only 3 of the urban areas had wetlands as a barrier.

The remaining barriers, mountains and protected lands, also produced lower mean proximities for the urban areas affected. However, the differences in the means

Table 7. Mean Values of Proximity Index for Urban Areas With and Without Various Barriers to Urban Area Expansion, 1950 and 2010.

Barrier	Mean for Areas with Barrier	Mean for Areas Without Barrier	Difference in Means
Proximity Index in 1950			
Water	0.68	0.81	-0.13 ***
Wetlands	0.64	0.79	-0.15 *
Mountains	0.75	0.79	-0.03
Protected Land	0.73	0.79	-0.06
Proximity Index in 2010			
Water	0.65	0.79	-0.14 ***
Wetlands	0.65	0.77	-0.12
Mountains	0.71	0.77	-0.07
Protected Land	0.70	0.77	-0.07 *

*Significant at the 0.05 level

***Significant at the 0.001 level

between the areas with and without barriers were less, 0.07 in 2010 and even less than that in 1950. Only the effect of protected lands in 2010 was statistically significant.

The effect of water as a barrier on proximity was high in both years while the effect of mountains and protected lands seemed to increase somewhat from 1950 to 2010. This may make some sense. Cities with water as a barrier would have been started at the coastline; they were often founded to serve as ports. Thus the barrier would have immediately affected the shape of the area as it developed. Cities with mountains or protected lands as barriers were not necessarily located initially adjacent to those barriers. In fact they most likely were not. It was as the urban areas expanded that they encountered the barriers, which then had the effect of altering the directions of further development. This makes it reasonable that these barriers would have had a greater effect on the shape and the proximity index in later years.

Some of the urban areas are impacted by multiple barriers to development. Multiple regression models can be used to examine the combined effect of the barriers on proximity values. However a problem arises in using all 4 together in predicting proximity. A very high degree of overlap exists between the mountains and protected lands barriers. Nine urban areas are affected by both types of barriers. Significant areas of mountain are also protected lands. Only 3 of the 12 urban areas with protected lands are not also on the mountains list, and only 1 of the 10 urban areas with mountains is not also affected by protected lands. When both of these are included in a model, a very high degree of multicollinearity exists between the 2 variables. This adversely affects the coefficient estimates. As a result, it is appropriate to exclude one of the barriers from the model. Because the number of areas affected by protected lands is slightly larger and because it produced the only statistically significant difference in means, protected lands is included and mountains is excluded.

The results for the regression models using the barriers to predict proximity in 1950 and 2010 are presented in Table 8. Overall performance was quite reasonable, with R^2values of 0.29 and 0.32 in 1950 and 2010 respectively, both of which were statistically significant at the 0.001 level. For 2010, the 3 barrier measures accounted for virtually one-third of the variation in proximity among the urban areas.

As expected given the examination of the mean differences, water as a barrier had the greatest effect on proximity and was highly statistically significant in both years. In both years, the water barrier produced a predicted proximity that was 0.12 lower than for the areas not so affected. The estimated effect of wetlands was next highest, with coefficients of -0.09 and -0.05 in the 2 years. That these were lower than the

Table 8. Regression Models Predicting Proximity Index in 1950 and 2010 Using Multiple Barriers to Urban Area Expansion.

Barrier Independent Variable	Dependent Variable	
	Proximity Index 1950	Proximity Index 2010
Water	-0.117 ***	-0.122 ***
	(0.032)	(0.030)
Wetlands	-0.089	-0.048
	(0.059)	(0.054)
Protected Lands	-0.011 *	-0.022 *
	(0.033)	(0.031)
Constant	0.814 ***	0.794 ***
	(0.015)	(0.013)
R^2	0.288 ***	0.324 ***

*Significant at the 0.05 level

***Significant at the 0.001 level

differences observed for the means and were not statistically significant was not surprising. Not only are there only 3 urban areas with wetlands as a barrier, but 2 of the 3 have water as well and 1 of the areas, Miami-Fort Lauderdale-West Palm Beach has all 3. Protected lands predicted much smaller effects on proximity, decreases of 0.01 and 0.02. Half of the areas having protected lands as a barrier also are impacted by water, reducing the net effect of protected lands.

The Effect of Shape on Urban Development

Differences in the shapes of urban areas as measured by the proximity index can affect patterns of development within those areas. A less compact shape, a lower level of proximity, might be expected to increase the density of residential development. The argument for this is as follows: People value accessibility to the CBD and are willing to pay more for locations closer to the CBD. To the extent that there is less land closer to the CBD in an area with lower proximity than in a more compact area, the available supply in those locations is reduced. Having a reduced supply without change in the demand will result in increased prices for the land. With higher prices, people will choose to purchase less, which means the density of development is higher.

This can be investigated very directly. Table 9 shows the correlations between the proximity index and housing unit density for the urban areas for each of the 7 years. As expected, the relationship was negative, with the correlation coefficients from 1970 on being statistically significant. This consistency in the relationship is more remarkable

Table 9. Correlations of Proximity Index with Housing Unit Density in Urban Area.

Year	Correlation with Housing Unit Density
1950	-0.21
1960	-0.24
1970	-0.31 *
1980	-0.34 **
1990	-0.37 **
2000	-0.35 **
2010	-0.31 *

*Significant at the 0.05 level

**Significant at the 0.01 level

because of the large changes in density that were occurring in many of these urban areas over this 60-year period. The mean number of housing units per square mile dropped from 1,268 in 1950 to 1,080 in 2010. But this does not show the more extreme changes in density that occurred in some of the areas. The maximum density declined from 2,975 to 1,928, with the New York area being displaced by Los Angeles as the urban area with the highest density. The density of New York dropped by over 1,100 housing units per square mile while Los Angeles' density increased by over 300 (Ottensmann 2015). In spite of all this change, the negative relationship between proximity and density remained relatively consistent.

A more fine-grained consideration of the effects of proximity on density comes from looking at the variation in patterns of density within the urban areas. It has long been observed that the density of urban areas tends to decline as a negative exponential function of distance from the center. The standard monocentric model of urban economics provides an explanation (Muth 1969, Mills 1972). People desire accessibility to the CBD and are willing to pay more for locations closer to the center than farther away.[2] So the price of land will decline with distance from the center. However people also desire more space in their residential environment, more land and lower densities. And they can afford to purchase more land farther from the center given the lower price of land. This makes residential choice a tradeoff between greater accessibility to the center and more space. At equilibrium land prices and densities will both decline with distance from the CBD. With some not unreasonable assumptions regarding the production function for housing and elasticities, it can be shown that density will decline as a negative exponential function of distance from the center.

The mathematical equation for the negative exponential decline of density can be stated as follows:

$$D_i = D_0 e^{-\beta s_i}$$

where D_i is the density for each small area (such as a census tract) i and s_i is the distance from that tract to the CBD. D_0 is the central density, the density when the distance is 0, and β is the density gradient, the rate of decline of density with distance from the center.

[2] The original formulations of the model assumed everyone worked in the CBD and sought to minimize their transportation costs. With the increased decentralization of employment and the emergence of large centers of employment outside the CBD, the initial assumption no longer holds. Studies have shown that measures of accessibility to all of the employment in an urban area better predict densities within the areas than distance from the center (Song 1994, 1996; Ottensmann 2008). However the CBD remains the location of maximum accessibility to employment, so it remains reasonable that people would desire locations closer to the CBD.

These two values are parameters to be estimated. Finally, *e* is the base of the natural logarithms. The equation is typically estimated by taking the log of both sides of the equation to transform it into a linear equation which can be estimated using standard ordinary least squares regression. The parameters that are estimated are then transformed to provide the values for the density gradient and the central density.

The effect of an irregularly shaped urban area with lower proximity is likely to be experienced most greatly closer to the periphery where the effects of the irregular boundary would be greater. This would tend to cause greater increases in densities in those more distant areas. Density would not be declining as rapidly with distance from the center and the density gradient would be lower. So one might expect lower proximity values to be associated with smaller density gradients.

Higher land prices and densities would be expected to produce an overall increases in densities, already observed, which might push up the entire density curve and produce higher central densities. So lower proximity value could be associated with higher central densities.

A final hypothesis is more speculative. More irregular urban areas with lower levels of proximity may have greater variations in patterns of density within the areas. This could result in the negative exponential model doing a poorer job of predicting density and therefore a lower coefficient of determination R^2.

The urban patterns data have been used to estimate the parameters for the negative exponential decline of housing unit density for each year. Note that these estimates have been made only for the 42 urban areas with a single CBD. It is not clear how one might get single estimates of these values for areas with multiple CBDs and distances to those different centers. The results showed major declines in the average density gradient, central density, and coefficient of determination R^2 over the period from 1950 to 2010 (Ottensmann 2016). The decline in the density gradient over time has been the subject of the greatest research attention associated with the negative exponential model.

Table 10 provides the correlations of the proximity index values with the density gradient, central density, and coefficient of determination R^2 for each year. Starting with the density gradient, the correlations were positive, substantial (the smallest was 0.39, the largest 0.59), and statistically significant at least at the 0.01 level for every year. As the shape of urban areas become more irregular and proximity declines, the density gradient drops as well, as expected. So the first hypothesis is clearly supported. All of the correlations with central density were negative but were not statistically significant. All but 1 were in the narrow range from 0.16 to 0.21, so there was clearly consistency, even as some areas were experiencing large drops in their central densities. This can be seen as weak support for the suggestion that central densities would increase for urban areas with lower proximity index values.

Table 10. Correlations of Proximity Index with Parameters Estimated for Model Predicting the Negative Decline of Housing Unit Density with Distance from the Center for 42 Urban Areas with a Single Center.

Year	Correlation with Density Gradient	Correlation with Central Density	Correlation with Coefficient of Determination R^2
1950	0.45 **	-0.10	0.23
1960	0.39 **	-0.19	0.17
1970	0.59 ***	-0.21	0.34 *
1980	0.55 ***	-0.16	0.26
1990	0.47 **	-0.19	0.12
2000	0.45 **	-0.18	0.13
2010	0.43 **	-0.18	0.12

*Significant at the 0.05 level

**Significant at the 0.01 level

***Significant at the 0.001 level

The correlations of proximity with the coefficient of determination R^2 were all positive. They varied in size and only 1 was statistically significant. So definitely not inconsistent with the idea that the negative exponential model would predict densities more accurately in more compact areas with higher proximities and would perform less well in the more irregular, low proximity areas.

Conclusions

It makes sense that barriers including water, wetlands, mountains, and protected lands would affect the expansion of urban areas, producing areas that were less compact. The proximity index was able to successfully measure this effect. Estimates of the effects of the barriers on proximity were estimated.

Perhaps the most interesting findings were the effects of the proximity index on urban area density. The effect of shape on the overall density of urban areas was highly significant and consistent. Less compact urban areas with lower proximity index values tended to have higher overall density. In addition, shape affected patterns of density within the urban areas. The rate of decline of density with distance from the CBD estimated using the negative exponential model was smaller for urban areas with lower proximity values. And while the relationships were not as strong, for less compact areas the estimated central density tended to be higher and the fit of the negative exponential model lower.

The proximity index works as a measure of the compactness of the shape of urban areas. The previous paper (Ottensmann 2021) found the index values to be highly consistent with intuitive assessments of urban area shape. The findings that barriers to urban expansion were significantly related to this measure of shape and that the proximity index was significantly associated with various measures related to the density within urban areas reinforces that the index is capturing a meaningful quality of the urban areas.

The extent to which the shape is compact is only one property of the shape of an urban area that may be of importance. The degree that the boundary of the urban area is irregular versus smooth can also be significant, perhaps as one measure of urban sprawl. The measurement of such irregularity and its relationship to other aspects of the urban area can be a subject for further research.

References

Angel, Shlomo, Jason Parent, and Daniel L. Civco. 2010. Ten compactness properties of circles: measuring shape in geography. *Canadian Geographer* 54, 4 (Winter): 441-461.

Galster, George, Royce Hanson, Michael R. Ratcliff, Harold Wolman, Stephen Coleman, and Jason Freihage. 2001. Wrestling sprawl to the ground: defining and measuring an elusive concept. *Housing Policy Debate* 12, 4: 681-717.

Hammer, Roger B., Susan I. Stewart, Richelle L. Winkler, Volker C. Radeloff, and Paul R. Voss. 2004. Characterizing dynamic spatial and temporal residential density patterns from 1940-1990 across the North Central United States. *Landscape and Urban Planning* 69, 2-3 (August): 183-199.

Maceachren, Alan M. 1985. Compactness of geographic shape: comparison and evaluation of measures. *Geografiska Annaler. Series B, Human Geography* 67, 1: 53-67.

Mills, Edwin S. 1972. *Studies in the Structure of the Urban Economy*. Baltimore: Johns Hopkins Press for Resources for the Future.

Muth, Richard F. 1969. *Cities and Housing: The Spatial Pattern of Urban Residential Land Use*. Chicago: University of Chicago Press.

Ottensmann, John R., Seth Payton, and Joyce Man. 2008. Urban location and housing prices within a hedonic model. *Journal of Regional Analysis & Policy* 38, 1: 19-35. Available at *http://www.jrap-journal.org*.

Ottensmann, John R. 2014. Urban patterns dataset description. Available at *https://urbanpatternsblog.files.wordpress.com/2016/12/urban-patterns-dataset-description.pdf*.

Ottensmann, John R. 2015. Density of large urban areas in the U.S., 1950-2010. Available at *https://urbanpatternsblog.files.wordpress.com/2016/12/density-of-large-urban-areas.pdf*.

Ottensmann, John R. 2016. The negative exponential decline of density of large urban areas in the U.S., 1950-2010. Available at *https://urbanpatternsblog.files.wordpress.com/2016/12/negative-exponential-density-decline.pdf*.

Ottensmann, John R. 2017. On the choice of Combined Statistical Areas. Available at *https://urbanpatternsblog.files.wordpress.com/2017/04/combined-statistical-areas.pdf*

Ottensmann, John R. 2021. Measuring the shape of urban areas. Available at *https://urbanpatternsblog.files.wordpress.com/2021/02/measuring-urban-area-shape.pdf*.

Paulsen, Kurt. 2014. Geography, policy or market? New evidence on the measurement and causes of sprawl (and infill) in US metropolitan regions. *Urban Studies* 41, 12 (September): 2629-2645.

Radeloff, Volker C., Roger B. Hammer, Paul R. Voss, Alice E. Hagen, Donald R. Field, and David J. Mladenoff. 2001. Human demographic trends and landscape level forest management in the northwest Wisconsin Pine Barrens. *Forest Science* 47, 2 (May): 229-241.

Radeloff, Volker C., Roger B. Hammer, and Susan I. Stewart. 2005. Rural and suburban sprawl in the U.S. Midwest from 1940 to 2000 and its relation to forest fragmentation. *Conservation Biology* 19, 3 (June): 793-805.

Song, Shunfeng. 1994. Modelling worker residence distribution in the Los Angeles region. *Urban Studies* 31, 9 (November): 1533-1544.

Song, Shunfeng. 1996. Some tests of alternative accessibility measures: a population density approach. *Land Economics* 72, 4 (November): 474-482.

Theobald, David M. 2001. Land-use dynamics beyond the urban fringe. *Geographical Review* 91, 3 (July): 544-564.

U.S. Bureau of the Census. 1983. *1982 Economic Censuses. Reference Series. Geographic Reference Manual.* EC82-R-1. Washington, DC: U.S. Government Printing Office, Table 9, pp. 309-322.

U.S. Bureau of the Census. 2002. Urban area criteria for the 2000 Census. *Federal Register* 67, 51 (Friday, March 15): 11663-11670.

U.S. Bureau of the Census. 2011. Urban area criteria for the 2010 Census. *Federal Register* 76, 164 (Wednesday, August 24): 53030-53043.

U.S. Bureau of the Census. 2012. Census block shapefiles with 2010 Census population and housing unit counts. Downloaded from *http://www.census.gov/geo/www/tiger/tgrshp2010/pophu.html* on September 29, 2012.

U.S. Bureau of the Census. 2013. Core Based Statistical Areas (CBSAs) and Combined Statistical Areas (CSAs) 2013 delineation file. February 2013. Downloaded from *https://www.census.gov/population/metro/data/def.html* on February 13, 2014.

Urban Institute and Geolytics. 2003. *Census CD Neighborhood Change Database (NCDB): 1970-2000 Tract Data*. Somerville, NJ: Geolytics, Inc.

Appendix: Urban Areas with Barriers to Urban Expansion

Water as a Barrier
Boston-Providence
Buffalo
Chicago
Cleveland-Akron
Los Angeles
Miami-Fort Lauderdale-West Palm Beach
Milwaukee
New York
Norfolk-Virginia Beach
San Diego
San Francisco-Oakland-San Jose
Seattle-Tacoma
Tampa-St Petersburg

Wetlands as a Barrier
Miami-Fort Lauderdale-West Palm Beach
New Orleans
Norfolk-Virginia Beach

Mountains as a Barrier
Albuquerque
Denver
El Paso
Las Vegas
Los Angeles
Phoenix
Salt Lake City-Ogden-Provo
San Diego
San Francisco-Oakland-San Jose
Tucson

Protected Lands as a Barrier
Albuquerque
Denver
Las Vegas
Los Angeles
Miami-Fort Lauderdale-West Palm Beach
Norfolk-Virginia Beach
Phoenix
Salt Lake City-Ogden-Provo
San Diego
San Francisco-Oakland-San Jose
Seattle-Tacoma
Tucson