

# Measuring the Shape of Urban Areas

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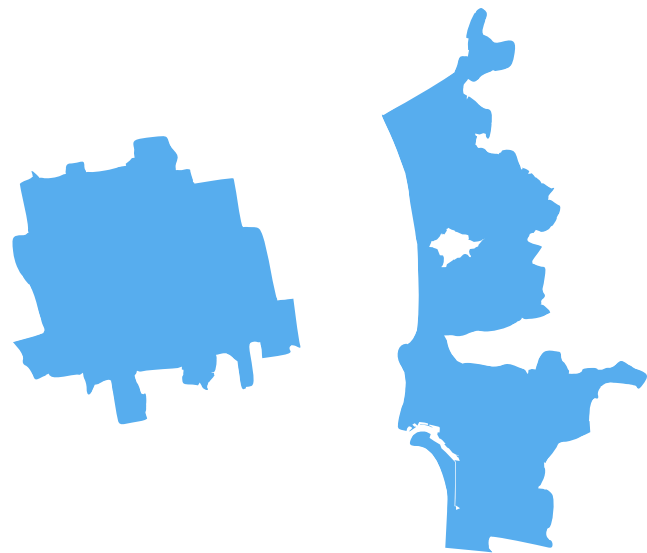
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## Abstract

Criteria are identified for a measure of shape compactness that would be appropriate for urban areas. A variety of types of measures that have been used for the shape of geographic areas are evaluated. The proximity index is selected. This is based on the mean distance from all points in the urban area to the Central Business District (CBD). The index is the ratio of the mean distance to the center of a circle having the same area as the urban area to the mean distance to the CBD for the urban area. The index ranges from 0 (least compact) to 1 (for a circular area). Values of the proximity index are calculated for 59 large urban areas in the United States in 2010 that had been defined using census tracts. Examination of the values and lists of the most compact and least compact urban areas and the maps illustrating the shapes of those areas shows the proximity index to be a reasonable measure of urban area shape.

## Introduction

The shapes of urban areas vary a great deal—obviously. Some areas are quite compact while others can be very irregular and elongated. The two urban areas shown in Figure 1 illustrate the variation. Columbus, OH, on the left, is almost a square while San Diego, on the right, both stretches out and has a very indented boundary. (These urban areas and all of the areas shown in blue in this paper are 2010 urban areas consisting of contiguous census tracts meeting a minimum density threshold that have been defined for my urban patterns research. More detail on the definition of these



**Fig 1. Columbus OH and San Diego urban areas.**

areas is provided later in this paper.)

Of interest are the forces that produce the variations in the shape of urban areas. These forces can potentially further affect development within those areas. And of course the shapes of areas will influence distances people have to travel within urban areas and this can affect other things, including patterns of development.

The shape of geographic areas has been of long-standing interest. Large numbers of measures of the shape of areas have been proposed. Numerous authors have reviewed the literature and these measures. Some examples include Frolov (1975), Hagget, Cliff, and Frey (1977), Montero and Bribiesca (2009), and Li, Goodchild, and Church (2013). Most useful is the article by Maceachren (1985), who provided a logical grouping and described many indices. He then proceeded to compare the performance of the indices in measuring the shapes of a sample of counties in the United States. Also noteworthy is an article by Angel, Parent, and Civco (2010) which took the unique approach of relating shape indices to various properties of a circle, seen as the most compact shape, arguing that the selection of an index should be based on the relevance of the property to the question under consideration.

Shape measures have been applied to urban areas in previous studies. Lo (1980) looked at the shape of Chinese cities. Longley, Batty, and Shepherd (1991) used a fractal measure to study changes in the shape of urban areas in the County of Norfolk in England. The question of whether greenbelts affected the shape of urban areas in South East England was the focus of a study by Longley, *et al.* (1992). Shape measures have likewise been used to examine other geographic questions. Griffith (1982) studied the relationship of the shape of an area to patterns of spatial interaction. Massam and Goodchild (1971) considered shape in relation to the efficiency of service areas. Related to this was a study of the shape of retail market areas by Simons (1974). Austin (1981) looked at the shapes of districts in West Malaysia.

Interest in the shape of geographic area extends beyond geography to the large body of literature devoted to the gerrymandering of legislative districts. The obvious features drawing attention are large, sprawling, irregular districts that represent the antithesis of compactness. It is therefore not surprising that many have turned to measures district shape to quantify the extent of gerrymandering. A few of the more widely cited articles in this literature include Young (1988), Niemi, *et al.* (1990), Polsby and Popper (1991), and Horn, Hampton, and Vandenberg (1993). I have, however, found these to be less useful than the works cited earlier.

Measures have been proposed for the shapes of geographic areas in general. In this paper I argue that the measurement of shape for urban areas raises particular issues that need to be addressed by any suitable measure of urban area shape, presenting in the next section criteria for a measure of urban area shape. The following section evaluates a range of commonly used measures of geographic shape, identifying those suitable for application to urban areas, and recommends the use of a proximity index.

The remainder of the paper illustrates the use of this index for the measurement of the shape of large urban areas in the United States in 2010. The data used, the definition of the urban areas, and the detailed specification of the index are presented, followed by illustrations of the performance of the index.

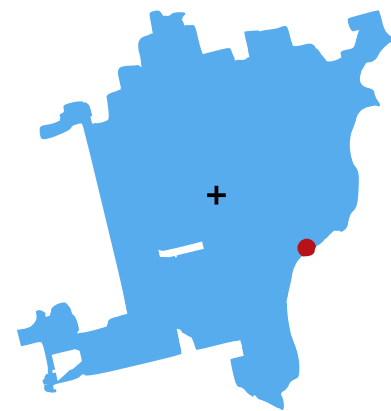
### Criteria for a Measure of Area Urban Shape

Three criteria are presented that should be met by any index to be used to measure the compactness of the shape of urban areas. These criteria are highlighted because unlike the more general criteria employed in evaluating shape indices, they are both of crucial importance for urban areas and a not met by many of the shape indices that have been proposed.

#### *Shape Relative to the Central Business District*

Urban areas have distinct, identifiable centers described as Central Business Districts (CBDs). These locations are important. Innumerable studies have shown that densities within urban area tend to decline in a regular fashion with distance from the CBD. The CBD is the focus of the classic monocentric model in urban economics (Muth 1969; Mills 1972), with the desire for accessibility to the CBD playing the key role.

A measure of the shape of an urban area should measure the compactness of that shape relative to the location of the CBD. Most shape indices that measure shape relative to a point location choose the location of the center of gravity or centroid of the shape. The difference is illustrated in the map of the Detroit urban area in Figure 2. The location of the CBD is show by the red dot, on the far eastern edge of the urban area. The center of gravity or centroid of the area is in the middle, of course, at the black cross. From the perspective of the centroid, the area would seem to be fairly compact, extending outward to roughly similar distances in all directions. But from the perspective of the CBD, the Detroit urban area looks far less compact. Portions of the urban area extend much farther from the CBD than they do from the centroid.



**Fig 2. Detroit urban area with Central Business District and area centroid (+).**

Several authors have raised the possibility of measuring distances and calculating measures of shape from the CBD rather than the center of gravity. Boyce and Clark (1964) presented the value of their index calculated using distances from the

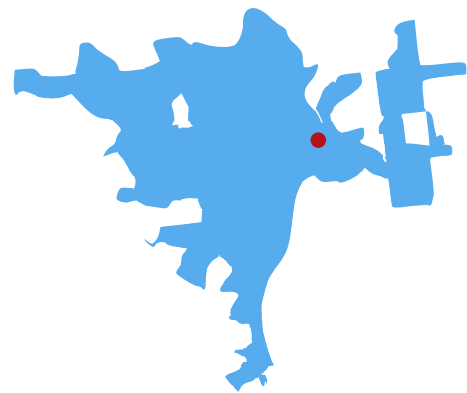
Milwaukee CBD along with the index using the area center of gravity. But curiously they say nothing about their rationale for choosing to include this. Angel, Parent, and Civco (2010) say that their proximity index can be calculated from any center, specifically noting that proximity is “a natural measure of accessibility of a metropolitan area to its Central Business District (CBD).” But then, in defining the index, they say that distance is to be calculated from the “Proximate Centre (which) is the centre of gravity of a shape.”

Some shape indices do measure shape relative to a specific point. For those, while the center of gravity or centroid is generally proposed as the point of reference, those indices can be adapted to measure shape relative to the CBD. Other shape indices make no reference to a central point, however, and they are therefore inappropriate for measuring the shape of urban areas.

#### *Deals with Holes, and Indentations, and Discontinuous Areas*

It is not uncommon for urban areas to have holes, areas that are completely surrounded by the urban area that are not considered to be part of the urban area. Urban areas can also have very irregular boundaries, with indentations that can produce problems with certain shape indices. And some urban areas can have multiple discontinuous shapes. The St. Louis urban area shown in Figure 3 illustrates the first two of these problems. The hole is obvious. But in addition, at least one shape index uses the distances from the center to points on the periphery in the calculation of the index. Consider radials extending east and northeast of the CBD. The large indentations in the boundary means that there would be gaps in the urban area as a line extended from the CBD to the farthest point on the periphery. The line would cross the perimeter of the urban area multiple times. This obviously creates ambiguity with respect to the measurement of distance to the periphery.

A further issue arises with urban areas that consist of multiple discontinuous shapes. An index suitable for urban areas should be able to accommodate such situations.

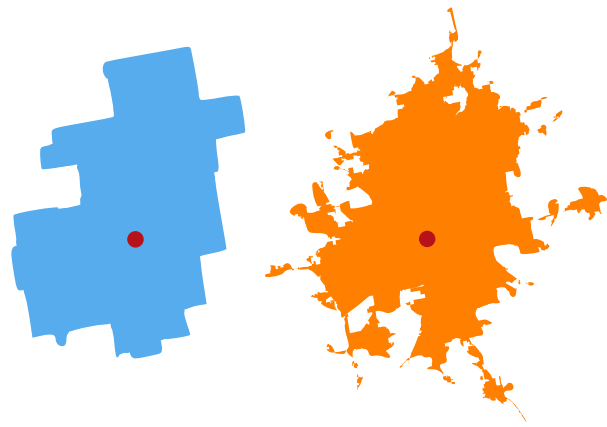


**Fig 3. St. Louis urban area holes and indentations.**

This final criterion is more general. The idea is that any measure of the shape of urban area should work well with and be practical to use for urban areas having the various forms in which they are typically defined. I see 3 quite different types of urban areas defined in different ways. First are urban areas that have been defined using data for small areas, generally from a census, that meet specified criteria including contiguity and minimum density. This is the approach I have taken to defining the urban areas for my urban patterns research that are considered in this paper. In my case, census tracts served as the building blocks. Angel (2012a) used a similar approach with census tracts for some of the urban areas he studied. Paulsen (2012) did much the same using block groups rather than tracts. This will result in urban areas with generally fairly smooth boundaries given the way the small areas have been defined, with exceptions most often coming when an irregular natural feature is used for a boundary. The map of the Indianapolis urban area on the left in Figure 4 is an example of this type of urban area.

The second type of urban area is the Urbanized Area defined by the United States Census. These are obviously important because they are the “official” areas. While they too are built up using small census areas, the nature of the definition produces very different urban areas. The first difference is that in addition to using census tracts or block groups (depending on the census), Urbanized Areas are also created using census blocks, which can be very small areas (literally blocks in built-up areas). Furthermore, in addition to using these smaller areas, the Urbanized Area definitions also provide for “hops” and “jumps” which allow the area to be extended across areas that do not meet the minimum density criterion to include more distant areas in the urban area (U.S. Bureau of the Census 2011). This can produce Urbanized Areas that have very irregular boundaries and that have long tendrils of urban development extending outwards for considerable distances. The map on the right in Figure 4 is the Indianapolis Urbanized Area, presenting a very different picture from the urban area shape on the left.

Finally are the urban areas derived from satellite imagery. The underlying data are for very small pixels in the image. In general, urban areas derived from such data will tend to show high degrees of irregularity, with a scattering of areas that are

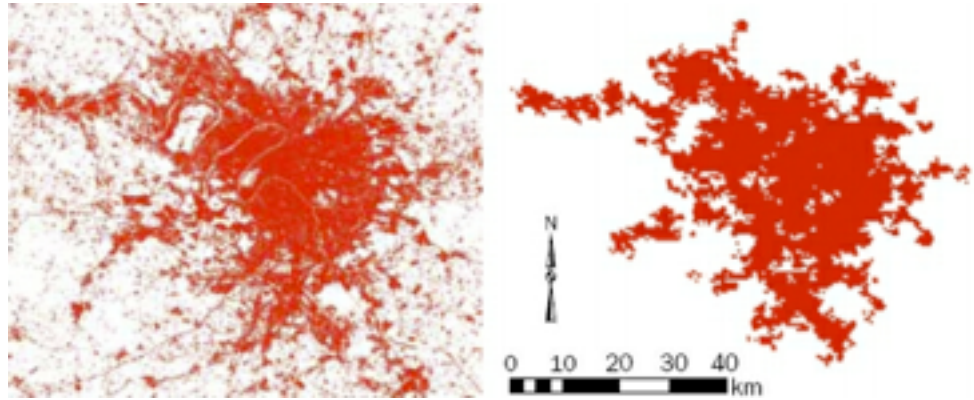


**Fig. 4. Indianapolis urban area and Urbanized Area.**

considered part of the urban area. But images from different satellite systems and differences in how the images are classified to define urban areas can vary greatly.

Figure 5 from Angel (2012a) shows 2 satellite-based maps of the

built-up area of Paris in 2000, the map on the left derived from Landsat images from NASA and the map on the right from Mod500 images, from Schneider, Friedl, and Potere (2009). Angel, *et al.* (2012b) provide an atlas with large numbers of satellite-based maps of urban areas throughout the world.



**Fig. 5. Two satellite-based maps of built-up area of Paris.**

Angel (2012). Original sources (left) NASA and (right) Schneider, Friedl, and Potere (2009)

## Evaluation of Specific Shape Measures

This section discusses various types of shape measures, loosely following the organization used by Maceachren (1985). The appropriateness of the indices for measuring the shape of urban areas is evaluated both with respect to the specific criteria laid out in the preceding section and more general considerations of the performance of the indices, many of which have been discussed by others.

### *Perimeter-area Indices*

Some form of the ratio of the perimeter of a shape to its area is likely the oldest and possibly the most commonly used measure of geographic shape. The rationale is simple and obvious. A circle is the most compact shape and has the smallest perimeter of any figure having the same area. The perimeter of an equal-area square is larger and that of a rectangle is larger still. So it seems obvious that comparing the perimeter to the area of a shape would form a reasonable basis for a measure of compactness.

A variety of specific formulations for an index have been proposed. Most have both terms of a ratio using the same units, either length or area. The index should keep the same value as the size of the shape is changed. Normalizing the index to a specific range such as 0 for least compact and 1 for most (a circle) can be done. One index would

be the ratio of the circumference of a circle having the area of the shape to the perimeter of that shape. Maceachren (1985) gives several examples.

The problem with using the perimeter of the shape is that a rough, jagged boundary can greatly increase the perimeter while having only minimal effect on the overall shape. Montero and Bribiesca (2009) gave an example in which the ratio of the perimeter squared to the area is increased by over 50 percent with only a minor degree of roughness added to the perimeter. Li, Goodchild, and Church (2013) found a perimeter index to be very sensitive to small changes in the coordinates of boundary vertices. And Wentz (2000) used as a measure of edge roughness the fractal dimension, which is a ratio using the logs of the perimeter and area.

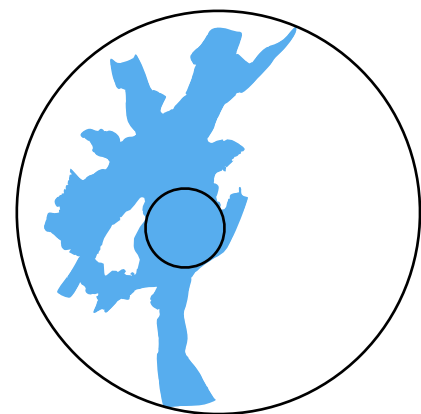
It is clear how this problem with perimeters that are not smooth becomes an issue for the shapes of urban areas. Raster (pixel) representations of urban areas immediately incorporate departure from smooth boundaries due to their use of square grid cells. And if the satellite-based urban area consists of numbers of separate areas or includes numbers of holes, the perimeter can be increased dramatically. An extreme problem arises with many Urbanized Areas. Consider the highly irregular boundary of the Indianapolis Urbanized Area shown in Figure 4. Its perimeter is extremely long and a product of the character of the boundary, not the general shape of the urban area.

Considering the other criteria, perimeter indices can accommodate holes, indentations, and discontinuous areas, though the presence of such features may well limit the utility of the indices as measures of the shape. But perimeter indices fail completely in representing the shape in relation to the CBD. Neither the perimeter nor the area of a shape are related to the location of any specific point within the shape. The location of the CBD is irrelevant.

### *Circumscribing and Inscribed Circles*

A second group of shape indices are related to the diameters or areas of multiple circles: The smallest circle circumscribing the shape, the largest circle that can be inscribed completely within the shape, and the circle having the same area as the shape. The smallest circumscribing circle and the largest inscribed circle for the Birmingham urban area are shown in Figure 6. Ratios of any pair of these diameters or areas could be used as a measure of the shape of an urban area.

The problem with these indices is that any measure that employs an extreme value—a minimum or a maximum—to characterize a distribution or a shape is completely sensitive to any variation in that



**Fig. 6. Birmingham urban area with circles circumscribed around and inscribed within.**

single extreme value. Consider the example of Birmingham. Lengthening or shortening one of the areas extending out from the body of the urban area would change the diameter of the circumscribing circle, possibly by a large amount, while having only limited effect on the area's overall shape. For the inscribed circle, increases or decreases in an indentation limiting the size of the circle would likewise have a significant effect. So would making changes to the extent of the hole bordering that circle. In fact eliminating the hole would just about double the size of the circle that could be inscribed within the area.

These measures can deal with holes, though as can be seen with Birmingham, the presence of a hole can have a disproportionate effect on the value of any index using the size of the largest inscribed circle. For some discontinuous urban areas, the locations of the separate areas might dramatically increase the size of the smallest circumscribing circle. And Urbanized Areas with long tendrils extending far from the main body of the shape could have very large circumscribing circles.

As is obvious with the Birmingham example, the locations of these circles and hence their dimensions are not dependent on the locations of the CBD. The centers of the circumscribing and inscribing circles are far from one another. Indeed, the center of the circumscribing circle looks to be very near the edge of both the inscribed circle and the Birmingham urban area itself.

#### *Distances to the Perimeter*

The distances from the center of a circle to all points on the boundary are equal. Indeed, that is frequently taken as the definition of a circle. Any shape that varies from a circle, that is more irregular and less compact, will have varying distances from a center to different points on the periphery. A measure of that variation can be used as a measure of the (non)compactness of the shape.

This is the first type of shape measure considered here where the shape is measured with respect to the location of a center. And while the center of gravity is typically used, the CBD could just as easily be used as the location from which distances to points on the periphery are measured. Indeed, Boyce and Clark (1964) gave the example of doing just that in calculating a shape measure for the Milwaukee area.

This is an index where its evaluation is dependent on the details of the implementation. In general terms, the description of the index refers to variation in the distances from the center to points on the periphery. However, there are an infinite number of points on the periphery. So how to calculate the distances and a value for the index in practice? The solution has been to create a set of equally spaced radials extending from the specified center to the edge of the shape, take the distances along each radial to the perimeter, and calculate a measure of the variation of those distances. It is obvious that where the radials intersect the boundary (and hence the distances) will

vary with the number of radials and the orientation of the entire set. The proposed solution is to minimize the problem by increasing the number of radials used. Tests have been conducted with varying numbers of radials using multiple orientations. Maceachren (1985) suggested that for the shapes he was considering, the index stabilized with between 15 and 40 radials, with slight fluctuations up to 120 radials. He used the latter number but suggested that in practice 40 to 50 would be sufficient “for most shapes.”

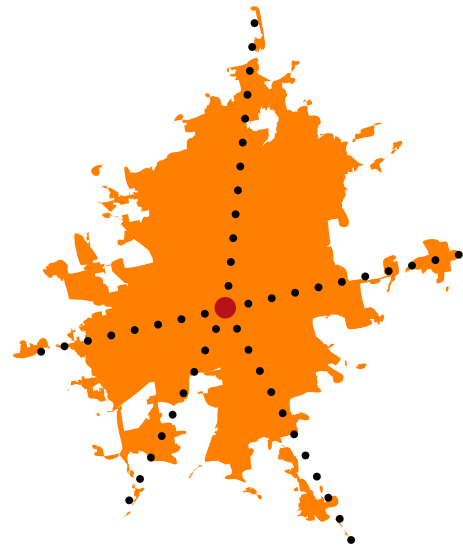
This uncertainty is slightly unsatisfying. What might be the issue in practice? Obviously it depends on the shape. For one of the regular urban areas such as the Indianapolis urban area on the left side of Figure 4, one would suspect this would not be a problem. But consider the Indianapolis Urbanized Area, which is displayed again in Figure 7, this time with dotted lines showing radials extending out along the longest tendrils extending from the body of the Urbanized Area. Note the narrow width of those tendrils. The slightest alteration in the orientation of radials used to measure distances to the periphery can make the difference whether the radial intersects the boundary at the most distant point at the end of the tendril or at a point far closer to the center.

This is hardly the only problem with this measure of shape. What happens if the radial crosses a hole? Maceachren (1985) suggested adding to the distance to the periphery the length of the crossing of the hole. This seems pretty arbitrary. Indentations where a radial crosses out of the urban area and reenters present a similar problem. And how could discontinuous portions of an urban area be accommodated? While this index seems as if it could be an attractive measure of compactness for reasonably simple geographic areas, the nature of different urban areas can present problems that make this index unsuitable for application in this context.

#### *Area within a Circle*

These next indices directly compare the area of a shape with a circle having the same area that is drawn around any designated center for the shape, which could be the CBD. These therefore can meet the criterion for indices that measure shape with respect to the CBD.

The key quantity for these measures is the area of that portion of the shape that lies within that circle, the intersection of the urban area with the circle. This is illustrated



**Fig. 7. Indianapolis Urbanized Area with radials extending to extreme points on periphery.**

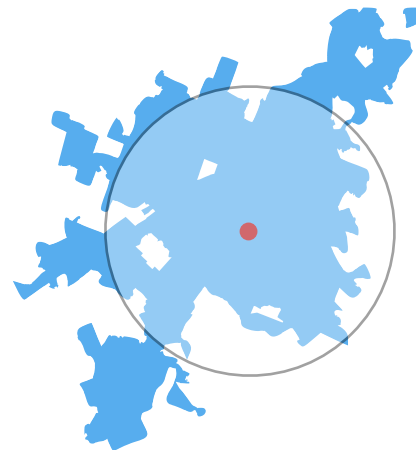
for the Philadelphia urban area in Figure 8. The circle has the same area as the urban area and is centered on the Philadelphia CBD. The area of intersection—the portion of the urban area within the circle—is shaded a lighter color.

Lee and Sallee (1970) first offered a shape index using the area of intersection. Their proposal was more general, for an index comparing any 2 shapes. They defined their index to be the ratio of the area of intersection of the shapes to the area of their union and called this a symmetric difference metric. As this more general measure, it had limitations, for they were less than precise about how one specified the relative sizes, position, and orientation of the shapes. Later applications of their idea have compared a shape to a circle having the same radius, as described here.

Because Lee and Sallee sought an index that measured the difference between 2 shapes and did not give priority to either shape, the idea of using the ratio of the area of intersection to the area of union made sense, as that produced a symmetric measure. However, as a measure of the compactness of a shape relative to a circle having the same area, it is less logical. The ratio of the area of intersection to the area of union is the ratio of the area of that portion of the shape that lies within the circle to the area of the circle, plus the area of the shape that falls outside. But since the area of the circle is by definition the area of the entire shape, the measure is then the ratio of the area of the shape inside to the total area of the shape plus the area of the shape outside. Including the area of the shape outside the circle in the denominator of the ratio twice is confusing. More reasonable would be to define an index that is the ratio of the area of intersection, the area of the shape inside the circle, to the total area of the shape. In other words, the proportion of the shape that falls within the circle. Or, to put it yet another way, the portion of the shape that is compact. This is a more straightforward and intuitive definition for a measure.

It has already been noted that this measure can satisfy the criterion of measuring the shape with reference to the CBD by positioning the comparison circle around that point. Holes, indentations, and discontinuous areas present no problem. If a hole or gap is within the circle, that is not area of the shape that is included within the circle. A discontinuous area in the circle would have its area included, while one outside would not.

The index can be readily computed for urban areas defined in different ways. For areas delineated using vector shapes, standard geographic information functions can be used to create the area of intersection and then find its area. For urban area that have



**Fig. 8. Philadelphia urban area with area of intersection of a circle with the same area.**

been created by aggregating small areas, one can take the distances from the centers of those areas to the CBD, select those where the distance is less than the radius of the circle having the same area, and sum their areas. The errors introduced from areas at the boundary being split by the circle tend to be balanced out as some areas will have their centers inside and some outside. This is generally a satisfactory procedure. With raster-based urban areas, pixels or cells within the circle can be selected, counted, and their total area determined.

While this is an acceptable measure for the shape of an urban area that meets the criteria, it is somewhat crude and not ideal. This can be seen by describing the construction of the index in a slightly different but equivalent way. Distances of small areas to the CBD are reclassified with a value of 1 if they are less than the radius of the circle and a value of 0 if they are farther way. The total area of the areas with values of 1 is then the area of intersection and is used to calculate the index. The reclassification of the continuous measure of distance to the CBD into a binary variable results in the loss of a great deal of information. For example, whether areas outside the circle are nearby or are more distant from the circle and the CBD is no longer known. Yet such differences would seem to be significant in measuring the compactness of the shape of the urban area. This provides the transition to the final indices being considered.

#### *Distances to Areas in the Shape*

These final indices use the distances to a center (which can be the CBD) from all areas or locations within the shape. These can be distinguished from the previous 2 types of indices as follows: The index using the distances along the radials considered distances only to points on the perimeter. These indices use distances to all points within the shape. The last index focusing on the area within the circle did use the distances of all points or areas to the center but then collapsed that information by considering only whether those distances were less than the radius of a circle having the same area as the shape.

Angel, Parent, and Civco (2010) defined their proximity index using the mean distances from all points in the shape to a central point. Using the CBD as the central point, they saw this as a natural measure of accessibility to the CBD. Obviously such a distance will generally increase with the size of the urban area. To formulate a measure of the compactness of the shape, one uses the mean distances of the points within a circle having an area equal to the shape to the center of that circle, which is  $2/3$  the radius of the circle. Then the index is the ratio of the mean distance for the circle to the mean distance for the shape. This varies from 0 to 1 for a circle.

An alternative is to use the mean of the squared distances to a center. This is the moment of inertia for rotation around an axis used in physics. A measure using this quantity was employed by Massam and Goodchild (1971) in a study of the efficiency of

regions used by a service agency and by Li, Goodchild, and Church (2013) for addressing the problem of defining regions. Both articles referred to the measure as the moment of inertia that is used in physics, though they provided no rationale as to why that would make it an appropriate measure for the measurement of shape. Note that this can likewise be standardized for size as the moment of inertia for a circular disk is  $1/2$  the area divided by  $\pi$ .

The two measures of shape yield similar results.<sup>1</sup> I would argue that the proximity measure using mean distance is slightly preferable to the moment of inertia using squared distances. Using distance squared gives greater weights to the longer distances for locations farther from the center. This would make the measure somewhat more sensitive to changes or errors at the boundary and especially sensitive to more distant points such as the tendrils associated with Urbanized Areas. In addition, the mean of the distance to locations within a shape is more intuitive than the mean of the squared distance.

Like the previous index, the proximity measure meets the criteria for a measure of the shape of urban area. Distances are to the CBD. Holes, indentations, and noncontiguous areas are not a problem as distances are to where the urban area is located and not to where it is not. And just as with the previous measure, the value of the index can be readily calculated for different types of urban areas using available capabilities. Mean distances for small areas can be calculated directly as an average of the distances weighted by their areas. Distances to a point can be calculated and aggregated for urban areas defined as rasters. Urbanized areas are aggregations of census blocks and larger areas which are themselves aggregations of census blocks. Distances to the CBD can be calculated for central points within census blocks. The block centers within the Urbanized Area can be selected and the area-weighted mean distance calculated. The detailed, formal presentation of the proximity index as used in the present investigation is given in the next section.

## Calculating Measures of Shape for Urban Areas

The ultimate objective of this section is the explanation of the manner in which the proximity index as a measure of the shape compactness of urban areas is calculated. The section begins with a description of the basic data and how it has been used to define the urban areas which are the subject of the investigation. Locations of the CBDs and the association of tracts with those centers is addressed next. The section concludes with the presentation of the proximity index formulated for this study.

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<sup>1</sup> In a simple test, I calculated an index using the moment of inertia for the 42 of my 59 urban areas with a single center and compared those values to the ones using the proximity index. The correlation between the 2 measures was 0.989, so there was little practical difference.

## *Defining the Urban Areas*

The current research is part of a larger research project that looks at patterns of development within 59 large urban areas in the United States from 1950 to 2010. These areas, the urban areas delineated for each census year, form the context within which the analysis is being undertaken. This section starts by identifying the urban areas included in the dataset, describes the housing unit data for census tracts that is the core of the data, and discusses how these data have been used to define urban areas from 1950 to 2010.

This research uses a dataset for the analysis of urban patterns over time that was developed with data on numbers of housing units in census tracts for large urban areas in the United States from 1950 to 2010. The tracts for urban portions of metropolitan areas were identified within the Combined Statistical Areas (CSAs) as delineated by the Office of Management and Budget for 2013 (U.S. Bureau of the Census 2013). CSAs were used rather than the more commonly employed Metropolitan Statistical Areas (MSAs) as it was felt they better represented the full extent of the metropolitan areas, including those instances in which 2 or more MSAs should more properly be considered to be parts of a single area (Ottensmann 2017). For those MSAs which were not incorporated into a CSA, the MSA was used.

The 59 CSAs and MSAs with 2010 populations over one million were selected for the creation of the dataset. A number of these areas had multiple large centers associated with separate urban areas that had grown together. This posed the issue of identifying those cases in which a second or third urban area could be considered sufficiently large in relation to the largest area to be included as an additional center around which urban development occurred. The decision was made by comparing the populations of census Urbanized Areas (either from the current census or the last census in which the areas were separate) with the largest area. An area was considered to be an additional center if its population were greater than 28 percent of the population of the largest area. The three areas included with the lowest percentages were Akron (with Cleveland), Tacoma (with Seattle), and Providence (with Boston). Areas with multiple centers have each center included in the name of the urban area.

The primary data source for this research was the Neighborhood Change Database developed by the Urban Institute and Geolytics (2003). This unique dataset provides census tract data from the 1970 through 2000 censuses, with the data for 1970 through 1990 normalized to the 2000 census tract boundaries. Population and housing unit data from the 2010 census were added by aggregating the counts from the 2010 census block data (U.S. Bureau of the Census 2012).

Housing units and housing unit densities—the numbers of housing units divided by the land areas of the tracts in square miles—are used in this research rather than the more commonly employed population and population density measures for two

reasons. Housing units better represent the physical pattern of urban development as they are relatively fixed, while the population of an area can change without any changes in the stock of housing. Other studies of urban patterns have made similar arguments for choosing housing units over population, for example Galster, *et al.* (2001), Theobald (2001), Radeloff, Hammer, and Stewart (2005), and Paulsen (2014).

Using housing units also allows the extension of the analysis to census years prior to 1970. The census provides data on housing units classified by the year in which the structure was built, and these data are included in the Neighborhood Change Database. The 1970 year-built data can be used to estimate the numbers of housing units present in the census tracts for 1940, 1950, and 1960. Several prior studies have used the housing units by year-built data to make estimates for prior years in this manner, though they have used more recent census data to make the estimates, not the earlier 1970 census data (Radeloff, *et al.* 2001; Theobald 2001; Hammer, *et al.* 2004; Radeloff, Hammer, and Stewart 2005).

Sources of error in these housing unit estimates for earlier years from the year-built data arise from imperfect knowledge of the year in which the structure was built and from changes to the housing stock due to demolitions, subdivisions, and conversions to or from nonresidential uses. These errors increase for estimates farther back in time. Numbers of housing units for 1970 to 1990 were estimated from the 2000 year-built data and compared with the census counts in the Neighborhood Change Database. The judgment was made that estimates 2 decades back involved acceptable levels of error, but this was not the case for 3 decades back. As a result, the decision was made to use the housing unit estimates for 1950 and 1960 but not for 1940.

Urban areas have been defined for the broader urban patterns research for each census year since 1950 consisting of those tracts contiguous to an urban center meeting a minimum housing unit density threshold. (This is comparable to the way in which the census defines Urbanized Areas using blocks and larger units and Paulsen (2012) defined urban areas using block groups.) For the definition of Urbanized Areas for the 2000 and 2010 censuses, a minimum population density of 500 persons per square mile was required for an area to be included (U.S. Bureau of the Census 2002, 2011). Using the ratio of population to housing units for the nation as a whole in both 2000 and 2010 of 2.34 persons per unit, a density of 500 persons per square mile is almost exactly equivalent to 1 housing unit per 3 acres or 213.33 units per square mile. This was used as the minimum urban density threshold. Note that this is a measure of gross density, not lot size, as the areas of roads, nonresidential uses, and vacant land are included.

To provide for a set of urban areas that represents the cumulative expansion of the urban areas over time, a further condition was imposed that if a census tract did not exceed the minimum housing unit density and had not been included in the urban area in any given year, it would not be included in urban areas delineated in earlier years even if the density exceeded the minimum. The rule has been imposed in this direction

—if rural, then not urban earlier—rather than in the opposite direction—if urban, then urban later—because the more recent data are considered to be generally more accurate.<sup>2</sup>

### *Identifying the Central Business Districts and Calculating Distances*

The location of the CBD must be specified to measure distances. One of the only efforts by the Census to do so came in a report for the 1983 economic censuses (U.S. Bureau of the Census 1983). This lists the census tracts comprising the CBD for many larger cities. This information was used to identify the CBD tracts for those urban areas included and for which the tract numbering and boundaries were the same for 2000. For the other urban centers, the tract or tracts for the CBD were identified by determining the location of the city hall or other major government buildings and examining the pattern of major roads, which generally converge on the CBD. The centroid of the CBD tract or tracts was taken as the center. Distances to the center were measured in miles from the centroids of each of the census tracts in the urban area.

For the 16 areas with 2 or 3 centers, the calculation of the distances from the census tracts to the CBD and the values of the proximity index required the partitioning of those urban areas. Each tract had to be assigned to one of the multiple CBDs. For those areas for which separate Urbanized Areas were delineated for the census in 2010, the tracts were assigned using the Urbanized Area boundaries. For the urban areas with a single Urbanized Area encompassing the entire urban area, tracts were assigned to the nearest center. In a few instances, modifications were made based on the geography of the area. For example, for Tampa-St. Petersburg, tracts on either side of Tampa Bay were assigned to the center on that side, even though in a few instances they were actually closer to the other center.

### *Calculating the Proximity Index*

The proximity index begins with the mean distance from all locations or areas within the urban area to the CBD. The mean distance of the points within a circle having the same area is then divided by this distance to produce the value of the proximity index. In the most general sense, the mean distance  $\bar{S}$  will be the integral of the distances over all locations within the shape:

$$\bar{S} = \int_A s da$$

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<sup>2</sup> More detail on the construction of the dataset and the delineation of the urban areas is provided in Ottensmann (2014).

where  $s$  is the distance from each infinitesimal area  $da$  in the shape to the CBD.

For an urban area composed of small areas such as census tracts, the areas being considered here, the mean distance will be the average of the distances from the tract centroids to the CBD weighted by the area of the tract:

$$\bar{S} = \frac{\sum_i s_i a_i}{\sum_i a_i} = \frac{\sum_i s_i a_i}{A}$$

where  $s_i$  is the distance from the center of tract  $i$  to the CBD,  $a_i$  is the area of tract  $i$ , and  $A$  is the total area of the urban area, the sum of the tract areas.

The mean distance to the points in a circle with the same area  $A$  as the urban area is  $2/3$  the radius of that circle, which is equal to the square root of the area divided by  $\pi$ . The proximity index  $P$  is then the ratio of this mean distance for the circle to the mean distance for the urban area:

$$P = \frac{2/3\sqrt{A/\pi}}{\bar{S}}$$

The value of  $P$  ranges from 0 to 1, with the value of 1 for a circular area, maximum compactness, and 0 for the least compact area. Note that the inverse of  $P$  is a measure of how much longer the mean distance to the CBD for the urban area is than the mean distance if that area were circular.

For those urban areas having 2 or 3 CBDs, the overall proximity index for the urban area is the average of the values of the index calculated for the areas of the tracts associated with each of those CBDs weighted by the area of those subareas. Let  $P_j$  be the proximity index for the area of the tracts associated with CBD  $j$  and let  $A_j$  be the area of those tracts. Then the proximity index  $P$  for the entire urban area is:

$$P = \frac{\sum_j P_j A_j}{\sum_j A_j}$$

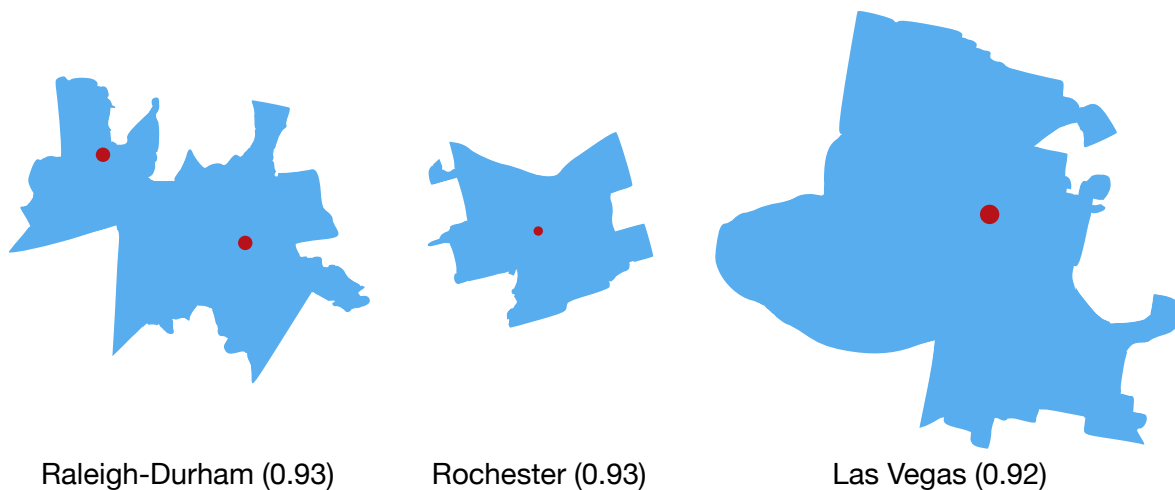
Since the values of the proximity indices for the subareas vary from 0 to 1, the value for the overall proximity index will have the same range, with a value of 1 only when each of the subareas is circular (very unlikely).

## Shapes of Urban Areas in 2010

To assess the performance and reasonableness of this proximity index as a measure of the compactness of the shape of urban areas, values for the index have been calculated for the 2010 urban areas created using census tracts as described above. Table 1 presents some basic summary statistics for the index for the 59 large urban areas. The mean for the proximity index is 0.76. For the average urban area, the mean distances from points within the area are about 1.3 times the mean distance, 30 percent more, than it would have been had the area been circular. Urban area proximity ranges from a minimum of 0.53 to a maximum of 0.93, a reasonable range indicating that the index is discriminating among the urban areas in the measurement of compactness. The urban areas are well spread out within this range and can be considered to be approximately normally distributed. The lowest values being just above 0.5 implies that mean distances in those areas are nearly twice the mean distances for circular areas.

**Table 1. Summary statistics**

<b>Statistic</b>	<b>Value</b>
Mean	0.76
Minimum	0.53
First quartile	0.69
Median	0.77
Third quartile	0.83
Maximum	0.93



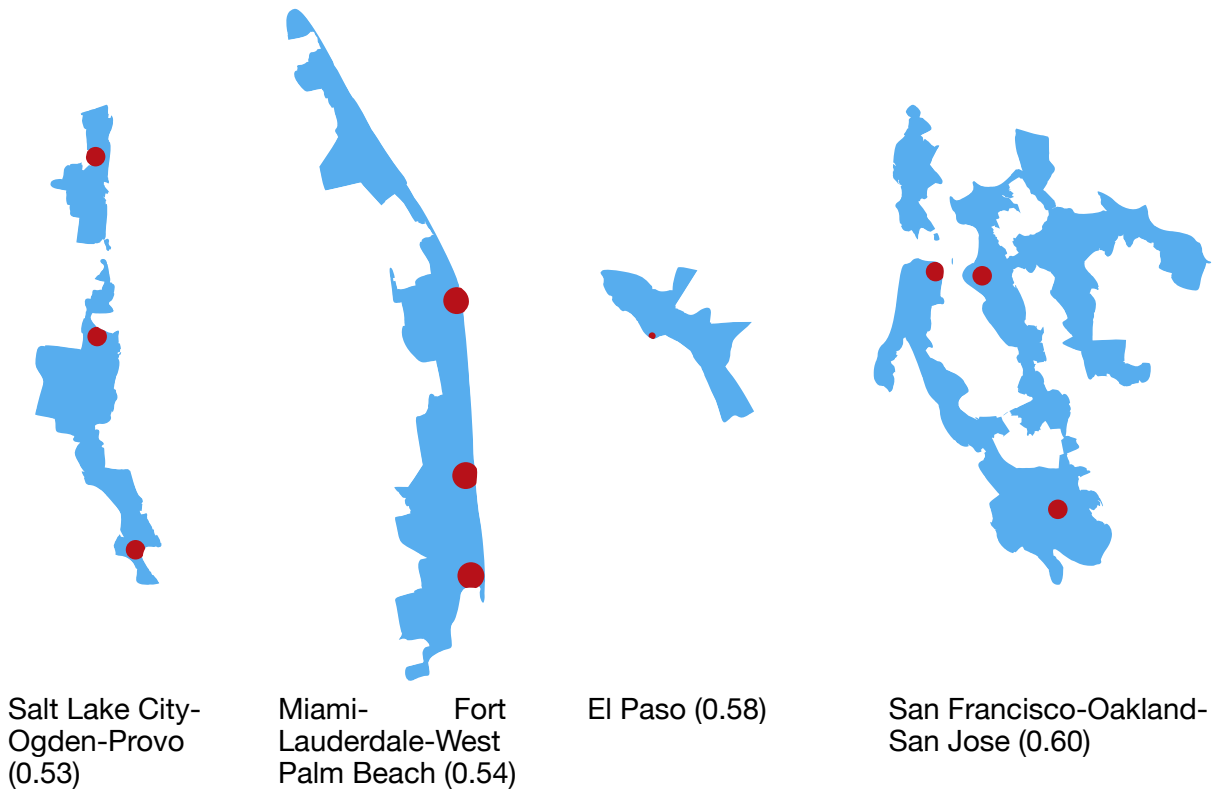
**Fig. 9. The three urban areas with the highest proximity values (index values in parentheses).**

It is useful to look at the urban areas that fall at the extremes. Starting with those that have the highest proximity index values, Table 2 gives the listing of the top 8. Figure 9 displays the shape of the 3 most compact areas, Raleigh-Durham (with 2 centers), Rochester, and Las Vegas.<sup>3</sup> Proximity index values for these areas ranged from 0.92 to 0.93. While visually discriminating compactness among these more compact shapes is difficult, it seems reasonable to conclude that they are examples of compact urban areas. One interesting point about another area on the list. Fourth was Columbus with a proximity index value of 0.91. Figure 1 at the beginning of this paper shows Columbus as an example of a more compact urban area. I had selected Columbus to serve as that example before having calculating the index values and knowing that it would be the fourth most compact urban area.

**Table 2. Areas with highest proximity index.**

Urban Area	Proximity Index
Raleigh-Durham	0.93
Rochester	0.93
Las Vegas	0.92
Columbus	0.91
Dayton	0.91
Indianapolis	0.89
Portland	0.89
Atlanta	0.87

<sup>3</sup> In this figure and other figures displaying 2 or more urban areas, the relative sizes of the areas in the figure are consistent with the relative sizes of the urban areas. However, for display purposes, areas are not all necessarily shown at exactly the same scale.



**Fig. 10. Four of the five urban areas with the lowest proximity values (index values in parentheses).**

Of course the urban areas at the other extreme, the least compact areas with the lowest proximity values, are the more interesting because they have the more irregular shapes. Table 3 lists the 8 areas with the lowest index values. Figure 10 show the shapes of 4 of the 5 least compact areas, Salt Lake City-Ogden-Provo, Miami-Fort Lauderdale-West Palm Beach, El Paso, and San Francisco-Oakland-San Jose. Index values ranged from 0.53 to 0.60. San Diego, the third least compact

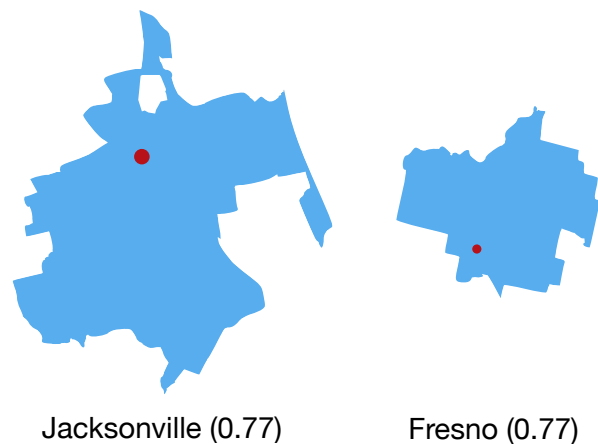
**Table 3. Areas with lowest proximity index.**

Urban Area	Proximity Index
Salt Lake City-Ogden-Provo	0.53
Miami-Fort Lauderdale-West Palm Beach	0.54
San Diego	0.54
El Paso	0.58
San Francisco-Oakland-San Jose	0.60
Sacramento	0.64
New York	0.64
Boston-Providence	0.64

area, was selected for Figure 1 as an example of an area that was not compact, again with the selection having been made prior to calculation of the index.

The 2 least compact areas, Salt Lake City and Miami, each have 3 centers and are very elongated areas. But it is important to remember that the value of the proximity index is not based on the overall length of these areas. Index values were computed for the areas associated with each of the centers, which were then weighted. In these cases, each of those subareas is still more or less elongated and not compact, so the overall measure reflects the lack of compactness of the component parts. El Paso is a smaller area, again elongated, and it is not difficult to see how its shape is not dissimilar to the shapes of the portions of the first two areas associated with each of their centers. The San Francisco Bay area can only be described as just plain highly irregular. That the New York and Boston-Providence urban areas also make the list of the least compact urban areas is hardly surprising given they they too are highly irregular.

So what does a typical urban area with an average degree of compactness look like? Figure 11 shows the shapes of 2 urban areas with proximity index values equal to the median index of 0.77, Jacksonville and Fresno. (Jacksonville was actually the median area, but the proximity for Fresno was virtually identical.) Jacksonville has a somewhat irregular boundary, though nothing like a San Francisco. As was the case with many of the less compact areas, the CBD was displaced from the center, also contributing to a lower proximity index. Fresno represents a different type of case. While the shape is hardly a circle, as an abstract shape it does not show the dramatic departures from compactness. But note the location of its CBD, very close to the edge of the urban area. It is not unreasonable to conclude that it is this extreme location of the CBD that is contributing to Fresno's having the median proximity as opposed to being considered more compact.



**Fig. 11. Urban areas with the median proximity value (index values in parentheses).**

## Conclusions

The proximity index is an effective measure of the compactness of the shape of urban areas. Using the mean distances from points within the area to the CBD, it is based on a quantity of substantive importance to urban areas rather than a more abstract geometric quality. The index meets the criteria identified for a shape index for urban areas. It makes maximum use of the distance information rather than collapsing it into a binary variable.

Examination of urban areas with higher and lower values for the proximity index suggests that the index is consistent with subjective judgements regarding the relative compactness of those areas. An interesting test of this, that had not been planned, came with the selection of the urban areas to serve as exemplars of compact areas and areas that were the opposite. I had selected Columbus, OH, and San Diego as the two areas for Figure 1 and had created that figure early in this work, prior to having calculated the values of the proximity index. The results showed Columbus to be the fourth most compact urban area with an index value only 0.02 less than the most compact area. And San Diego was the third least compact area, with an index only 0.01 more than the least compact area. So the proximity index values were highly consistent with the intuitive assessment made initially.

Of course the development of the index and the demonstration of its reasonableness is only a first step. One is interested in the shapes of urban areas and their measurement to investigate questions involving what types of factors might have produced variations in shape and how the shape affects other patterns within urban areas. This will be the focus of a subsequent paper.

## References

- Angel, Shlomo, Jason Parent, and Daniel L. Civco. 2010. Ten compactness properties of circles: measuring shape in geography. *Canadian Geographer* 54, 4 (Winter): 441-461.
- Angel, Shlomo. 2012a. *Planet of Cities*. Cambridge, MA: Lincoln Institute of Land Policy.
- Angel, Shlomo, Jason Parent, Daniel L. Civco, and Alejandro M. Blei. 2012b. *Atlas of Urban Expansion*. Cambridge, MA: Lincoln Institute of Land Policy.
- Austin, Robert F. 1981. The shape of West Malaysia's districts. *Area* 13, 2: 145-150.
- Boyce, Ronald R., and W. A. V. Clark. 1964. The concept of shape in geography. *Geographical Review* 54, 4 (October): 561-572.
- Griffith, Daniel A. 1982. Geometry and spatial interaction. *Annals of the Association of American Geographers* 72, 3 (September): 332-346.
- Frolov, Yu S. 1975. Measuring the shape of graphical phenomena: a history of the issue. *Soviet Geography* 16, 10: 676-687.

- Galster, George, Royce Hanson, Michael R. Ratcliff, Harold Wolman, Stephen Coleman, and Jason Freihage. 2001. Wrestling sprawl to the ground: defining and measuring an elusive concept. *Housing Policy Debate* 12, 4: 681-717.
- Haggett, Peter, Andrew D. Cliff, and Allan Frey. 1977. *Locational Methods*, second ed. New York: John Wiley.
- Hammer, Roger B., Susan I. Stewart, Richelle L. Winkler, Volker C. Radeloff, and Paul R. Voss. 2004. Characterizing dynamic spatial and temporal residential density patterns from 1940-1990 across the North Central United States. *Landscape and Urban Planning* 69, 2-3 (August): 183-199.
- Horn, David L., Charles R. Hampton, and Anthony J. Vandenberg. 1993. Practical application of district compactness. *Political Geography* 12, 2 (March): 103-120.
- Lee, David R., and G. Thomas Sallee. 1970. A method of measuring shape. *Geographical Review* 60, 4 (October): 555-563.
- Li, Wenwen, Michael F. Goodchild, and Richard Church. 2013. An efficient measure of compactness for two-dimensional shapes and its applications in regionalization problems. *International Journal of Geographical Information Science* 27, 6: 1227-1250.
- Lo, C. P. 1980. Changes in the shape of Chinese cities. *Professional Geographer* 32, 2 (May): 173-183.
- Longley, Paul A., Michael Batty, and John Shepherd. 1991. The size, shape and dimension of urban settlements. *Transactions of the Institute of British Geographers* 16, 1: 75-94.
- Longley, Paul, Michael Batty, John Shepherd, and Graham Sadler. 1992. Do green belts change the shape of urban areas? A preliminary analysis of the settlement geography of South East England. *Regional Studies* 26, 5: 437-452.
- Maceachren, Alan M. 1985. Compactness of geographic shape: comparison and evaluation of measures. *Geografiska Annaler. Series B, Human Geography* 67, 1:53-67.
- Massam, Bryan H., and Michael F. Goodchild. 1971. Temporal trends in the spatial organization of a service agency. *Canadian Geographer* 15, 3 (September): 193-206.
- Mills, Edwin S. 1972. *Studies in the Structure of the Urban Economy*. Baltimore: Johns Hopkins Press for Resources for the Future.
- Montero, Raul S., and Ernesto Bribiesca. 2009. State of the art of compactness and circularity measures. *International Mathematical Forum* 4, 27: 1305-1335.
- Muth, Richard F. 1969. *Cities and Housing: The Spatial Pattern of Urban Residential Land Use*. Chicago: University of Chicago Press.
- Niemi, Richard G., Bernard Grofman, Carl Carlucci, and Thomas Hofeller. 1990. Measuring compactness and the role of a compactness standard in a test for partisan and racial gerrymandering. *Journal of Politics* 52, 4 (November): 1155-1181.
- Ottensmann, John R. 2014. Urban patterns dataset description. Available at <https://urbanpatternsblog.files.wordpress.com/2016/12/urban-patterns-dataset-description.pdf>.
- Ottensmann, John R. 2017. On the choice of Combined Statistical Areas. Available at <https://urbanpatternsblog.files.wordpress.com/2017/04/combined-statistical-areas.pdf>.
- Paulsen, Kurt. 2012. Yet even more evidence on the spatial size of cities: Urban expansion in the US, 1980-2000. *Regional Science and Urban Economics* 42, 4 (July): 561-568.
- Paulsen, Kurt. 2014. Geography, policy or market? New evidence on the measurement and causes of sprawl (and infill) in US metropolitan regions. *Urban Studies* 41, 12 (September): 2629-2645.

- Polsby, Daniel D., and Robert D. Popper. 1991. The third criterion: compactness as a procedural safeguard against gerrymandering. *Yale Law & Policy Review* 9, 2: 301-353.
- Radeloff, Volker C., Roger B. Hammer, Paul R. Voss, Alice E. Hagen, Donald R. Field, and David J. Mladenoff. 2001. Human demographic trends and landscape level forest management in the northwest Wisconsin Pine Barrens. *Forest Science* 47, 2 (May): 229-241.
- Radeloff, Volker C., Roger B. Hammer, and Susan I. Stewart. 2005. Rural and suburban sprawl in the U.S. Midwest from 1940 to 2000 and its relation to forest fragmentation. *Conservation Biology* 19, 3 (June): 793-805.
- Schneider, Annemarie, Mark A. Friedl, and David Potere. 2009. A new map of global urban extent from MODIS data. *Environmental Research Letters* 4, 4 (December): 1-11.
- Simons, Peter L. 1974. Measuring the shape distortions of retail market areas. *Geographical Analysis* 6, 1 (October): 331-340.
- Theobald, David M. 2001. Land-use dynamics beyond the urban fringe. *Geographical Review* 91, 3 (July): 544-564.
- U.S. Bureau of the Census. 1983. 1982 *Economic Censuses. Reference Series. Geographic Reference Manual*. EC82-R-1. Washington, DC: U.S. Government Printing Office, Table 9, pp. 309-322.
- U.S. Bureau of the Census. 2002. Urban area criteria for the 2000 Census. *Federal Register* 67, 51 (Friday, March 15): 11663-11670.
- U.S. Bureau of the Census. 2011. Urban area criteria for the 2010 Census. *Federal Register* 76, 164 (Wednesday, August 24): 53030-53043.
- U.S. Bureau of the Census. 2012. Census block shapefiles with 2010 Census population and housing unit counts. Downloaded from <http://www.census.gov/geo/www/tiger/tgrshp2010/pophu.html> on September 29, 2012.
- U.S. Bureau of the Census. 2013. Core Based Statistical Areas (CBSAs) and Combined Statistical Areas (CSAs) 2013 delineation file. February 2013. Downloaded from <https://www.census.gov/population/metro/data/def.html> on February 13, 2014.
- Urban Institute and Geolytics. 2003. *Census CD Neighborhood Change Database (NCDB): 1970-2000 Tract Data*. Somerville, NJ: Geolytics, Inc.
- Wentz, Elizabeth A. 2000. A shape definition for geographic applications based on edge, elongation, and perforation. *Geographical Analysis* 32, 1 (January): 95-112.
- Young, H. P. 1988. Measuring the compactness of legislative districts. *Legislative Studies Quarterly* 13, 1 (February): 105-115.