Optimal Equilibrium Contracts in the Infinite Horizon with No Commitment Across Periods *

Subir K. Chakrabarti† and Jaesoo Kim‡

Department of Economics
Indiana University Purdue University Indianapolis(IUPUI)
425 University Blvd.
Indianapolis, IN 46202.

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Abstract: The paper studies equilibrium contracts under adverse selection when there is repeated interaction between a principal and an agent over an infinite horizon, without commitment across periods. We show the second-best contract is offered in a perfect Bayesian equilibrium of the infinite horizon model. Unlike the equilibrium contracts in the finite-horizon, the equilibrium contracts in the infinite horizon are not subject to either the *ratchet effect* or *take-the-money-and-run strategy*, but rely on a *carrot and stick* strategy. We study two important applications, one of which is about the optimal regulation of a publicly-held firm. This application has a mixture of both moral hazard and adverse selection. The other application is to the problem of optimal nonlinear pricing when the valuation of the buyers are drawn from a continuum.

Keywords: Adverse Selection, Games with incomplete information, Optimal Contracts with Commitment, Pooling Contracts, Separating Contracts, Perfect Bayesian Equilibrium, Optimal Contracts with No commitment.

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†email of Subir K. Chakrabarti : imxl100@iupui.edu

‡email of Jaesoo Kim : jaeskim@iupui.edu

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1 Introduction

We study equilibrium contracts in an infinite-horizon adverse selection model when there is no commitment across periods. In each period the principal offers a menu of contracts to the agent. The contract is valid only for the single period, and enforceable only in that period. At the beginning of each period, the principal updates beliefs based on the past history of contracts offered and choices made by the agent. As there is no commitment across periods, the principal can offer a contract based on these updated beliefs. This leads to the possibility that the past history can reveal information about the type of the agent. The agent recognizes this and takes this into consideration when choosing from the menu of contracts offered by the principal. Such calculations on the part of both the principal and the agent raise some very interesting issues about what kind of contracts are offered in equilibrium, whether any information is revealed in equilibrium, and if so in what way, and how quickly. It also raises questions about whether the principal can at all offer contracts that would lead the agent to quickly reveal the agent's type and what incentives, if any, the principal needs to offer in order to induce such a response from the agent.

The nature of contracts when there is no commitment across periods is of interest for several reasons, but two stand out. First, when there is no commitment across periods, there is the question of how the contracts evolve over time, especially given the potential for updating beliefs. Second, while enforceability of the contracts, and thus commitment is a fairly reasonable assumption in the short run, it is less likely to be the case that contracts are fully enforceable in the long run. For instance, when the interaction between a principal and an agent is repeated over many periods, it is quite likely that the principal offers a series of short run contracts[1].

Bolton and Dewatripont [2], Hart and Tirole [9], and Laffont and Tirole [11] and [12] have studied the dynamic adverse selection problem between a principal and an agent over a finite horizon, mostly in the case of finitely many types.[2] They have shown that if there is no commitment across periods, then there is the possibility that contracts reveal information about the type of an agent, and as the principal can use this information in the future, the ratchet effect comes into play. Further, since information rents may have to be paid in advance in period 1, some types of agent may have the incentive to use the *take the money and run strategy.* In the labor market for example, an employer has an

[1]This is especially the case when it involves buyers and sellers as the buyer can choose not to buy and there is less scope to write long term binding contracts.

[2]The dynamic moral hazard model with limited commitment has been studied by Radner [13], Radner [14], Spear and Srivastava [15], Rubinstein and Yaari [16] as well as others.

incentive to offer a tougher contract if she learns that the employee's productivity is high. Because of this ratchet effect, the high productivity employee would hesitate to reveal his/her type, and as a result, the employer may then have to offer a large compensation in the beginning. But this then leads to the problem that the less productive employee takes the offer meant for the high productivity employee, and leaves the employer after the first period. Therefore, equilibrium contracts are often pooling contracts in which types are not separated. This problem is especially acute when there is a continuum of types as shown in Laffont and Tirole [11].

Among the papers that study equilibrium contracts in which the time horizon is infinite and there is no commitment across periods, Battaglini [1] has some elements that are common with our result. In Battaglini [1] the type of the buyer can change from one period to the next in a Markov process. The paper shows that in the limit, as the types become persistent and constant, the equilibrium sequence of contracts becomes the single period second-best contract in each period like ours. The paper by Gerardi and Maestri [8] also studies equilibrium contracts when there is limited commitment across periods and the interaction can last over an infinite number of periods. They show that if the prior probability that the worker has low productivity is low then a pooling contract is offered. If the prior probability that the worker has low productivity is high, then the firm fires the unproductive worker. The result here shows that in a Perfect Bayesian equilibrium, the principal offers the second-best optimal contract in every period even after learning the type of the agent. Therefore, there is neither pooling nor termination of the contract in the perfect Bayesian equilibrium of our result. Thus the result we obtain here is distinct from that in Gerardi and Maestri [8].

As the single-period optimal contract (the second-best contract) usually separates types, the principal can infer the type of the agent at the end of the first period, if the agent chooses from the menu of contracts offered. The principal then updates beliefs and then offers a contract from period 2 onwards that is consistent with that belief. If the monotone hazard rate condition holds then in equilibrium the second-best contract is offered in period 1, after which the principal fully updates beliefs, and then offers the contract in the menu chosen by the agent in period 1 in the subsequent periods. The result shows that the equilibrium strategy is a "carrot and stick" strategy that makes it unprofitable for both the agent and the principal to deviate from it.

It is important to note at this point that even though the principal is fully able to infer the type of the agent in a separating contract, the principal does not use this information to *ratchet* up the terms of the contract in the following periods. When a separating contract is offered in period 1, the contract offered in all the subsequent periods retains all the information rent and the terms of the contract offered in period 1. Thus if the second-best contract is offered in period 1, then the contract offered in the future is the one the agent selected from the menu in period 1, together with any information rent that is part of the contract. This feature, that information rents will continue to be paid, is what induces the agent to pick an action consistent with its type. If this was not the case then the agent would have an incentive to conceal his/her type in order to prevent the principal from extracting the surplus from the agent in the future. The payment of the information rent in each period, which is consistent with the incentive constraints, prevents another potential problem that could arise in such situations. If all the rent is paid as a lump sum payment at the start, a less efficient agent may have the incentive to mimic the type of a more efficient agent, take the large lump sum payment and then leave. This strategy, often called the *take the money and leave strategy* becomes a possibility when all the information rent is paid in the beginning. Since the information rents of the second-best optimal contract are paid in each period, and not as a lump sum in the beginning, the problem of *take the money and leave strategy* does not arise in this case.

Therefore, when the interaction between the agent and the principal is over a long period, even when there is no commitment across periods, separating equilibrium contracts are such that neither the *ratchet effect*, nor the problem of *take the money and run strategy* arises, as the principal offers the same set of contracts to the agent in each period, even after learning the type of the agent. We believe that this is consistent with the observation that wage contracts in some occupations tend to be quite stable, for example for teachers, college professors and civil servants. It is also worth noting that these are jobs in which the employer and the employee both expect that the employee will be in the job for a fairly long period. Further, although the employer is quite likely to infer the productivity of the employee quite early, there is little evidence of the ratchet effect, nor is there evidence of a large compensation paid to the employee at the start of employment. The evidence also suggests that employees who reveal themselves to be a high-productivity type are likely to get a premium built into the wages or salaries; which can be interpreted as the information rent that is paid to the more productive types in the separating equilibrium contracts. Therefore, unlike the finite-horizon equilibrium contracts, in the infinite horizon, equilibrium contracts are not subject to either the *ratchet effect* or *take-the-money-and-run strategy*.

As already mentioned, the equilibrium strategies of the perfect Bayesian equilibrium analyzed here use a "carrot and stick strategy" in which deviations by the principal and the agent is deterred by the threat of credible punishments. The principal is deterred from deviating from the optimal contract, even after updating beliefs about the type of the agent, by the agent producing an output of zero for several periods. This is the "stick" part of the strategy. However, for the threat to produce zero output to be credible, the agent has to receive some payment back for carrying out the threat, as otherwise the payoff of the agent is zero in each period. Therefore, when the agent gets back to producing a positive output, after the punishment phase is over, the agent is given a slightly higher rent than before. This is the "carrot" part of the strategy. The strategy used by the principal to deter deviations by the agent is similar. The agent is deterred from deviating by the threat that the principal will offer a rent-free contract irrespective of the type of the agent for several periods, in case the agent deviates. Such "carrot and stick" strategies are similar to the ones used to analyze subgame perfect equilibrium of infinite horizon games of complete information as in Fudenberg and Maskin [6], as well as in an incomplete information setting as in Chakrabarti [3]. The result here shows that the general idea of such "carrot and stick" strategies can also offer useful insights about perfect Bayesian equilibrium in the case of adverse selection models by properly specifying the "stick" and the "carrot" and how these can be designed when the type of the agent is private information. Thus the result here shows that the general idea of the "carrot" and "stick" strategy can be adapted not only to the case of finitely many types as in Chakrabarti [3], but also to the case of the continuum of types in adverse selection models.

We show that the results can be widely applied by analyzing the model of regulation in Laffont and Tirole [11] in which the cost parameter β is drawn from the interval $[\underline{\beta}, \bar{\beta}]$.[3]. We also study the nonlinear optimal pricing model in which the type of the buyers are drawn from a continuum.

The main result is based on the continuum-type case but the result also holds for the discrete-type case. In fact the result for the discrete-type case can be quickly derived from the result for the continuum of type case. Our reason for focusing on the continuum of type case derives from the fact that the result is less obvious for the continuum of type case, and the fact that the result for the discrete-type case is an immediate corollary of the continuum of type case. In the case of the model studied by Laffont and Tirole [11], in which the type of the firm is drawn from a continuum, it is much harder for separation of types to hold in an equilibrium in the finite horizon, when there is a continuum of types, and pooling becomes the norm. This makes the result we present here of much

[3]It may be of interest to note that in the Laffont and Tirole [11], there is no possibility of any kind of separating equilibrium in the two-period model, indicating a particularly sharp contrast to the infinite horizon case.

greater interest and indicates the sharp contrast between the finite horizon models and the infinite horizon models. The paper is laid out as follows. In section 2 we describe the main definitions, the general framework and the main results. In section 3 we discuss the two main applications, with the infinite horizon version of the Laffont and Tirole model [11] in subsection 3.1 and the optimal nonlinear prices in subsection 3.2. In section 4 we conclude.

2 The Model

The main underlying model that we study here is the adverse selection model with a continuum of types in which the cost parameter of the agent, which is not known to the principal, varies over an interval. The principal knows the distribution of the cost parameter and offers a contract based on this information.

Let $[\underline{\theta}, \bar{\theta}]$ be the interval from which the cost of the agent is drawn and let $F(\theta)$ be the cumulative distribution function and $f(\theta) > 0$ be the density function on $[\underline{\theta}, \bar{\theta}]$. The second-best optimal contract[4] is then given by the menu of payment schedules $P(\theta)$ and production schedules $q(\theta)$ such that the pair $(P(.), q(.))$ maximizes

$$\int_{\underline{\theta}}^{\bar{\theta}} [S(q(\theta)) - P(\theta)] f(\theta) d\theta$$

subject to the incentive constraints

$$P(\theta) - \theta q(\theta) \geq P(\tilde{\theta}) - \theta q(\tilde{\theta})$$

for any $(\theta, \tilde{\theta})$ in $[\underline{\theta}, \bar{\theta}] \times [\underline{\theta}, \bar{\theta}]$, and the participation constraints

$$P(\theta) - \theta q(\theta) \geq 0$$

for all $\theta \in [\underline{\theta}, \bar{\theta}]$.

For the second-best optimal contract $(\hat{P}(.), \hat{q}(.))$, the output is given by

$$S'(\hat{q}(\theta)) = \theta + \frac{F(\theta)}{f(\theta)}. \tag{1}$$

We observe that as $F(\underline{\theta}) = 0$,

$$S'(\hat{q}(\underline{\theta})) = \underline{\theta}$$

[4]For a full and comprehensive discussion of the second-best optimal contract for the continuum of type case see [10], Appendix 3.1, pages 134 to 140.

and

$$S'(\hat{q}(\theta)) > \theta$$

for all $\theta > \underline{\theta}$.

The payment $\hat{P}(\theta)$ in the second-best optimal contract includes the cost of production and the information rent. Let $U(\theta)$ denote the information rent of type θ of the agent. Thus, the payment of agent of type θ is then

$$\hat{P}(\theta) = \theta\hat{q}(\theta) + U(\theta),$$

where the information rent of type $U(\theta)$ is given by[5]

$$U(\theta) = \int_{\theta}^{\bar{\theta}} \hat{q}(s)ds.$$

As $\hat{q}(\theta) > 0$ this shows that the information rent decreases monotonically as θ increases, and is zero for $\bar{\theta}$. That is the information rent tends to zero as the type θ tends to $\bar{\theta}$. Thus, the payment that an agent of type θ is offered in the second-best optimal contract is

$$\hat{P}(\theta) = \theta\hat{q}(\theta) + \int_{\theta}^{\bar{\theta}} \hat{q}(s)ds.$$

In the infinite-horizon adverse selection model, the principal and the agent negotiate a spot contract in each period given the past history. Thus in period t the principal offers a menu of contracts $m_t = (P_t(.), q_t(.))$. The agent then chooses a contract $(P_t(.), q_t(.))$ from this menu and then produces the output $q_t(.)$ associated with that contract. If the agent is of type θ, and chooses the contract $(P_t(.), q_t(.))$ in period t from the menu m_t, then the payoff of the agent is given by

$$U_t(\theta) = P_t(\theta) - \theta q_t(\theta).$$

The history up to period t is given by $h_{t-1} = \{(m_1, q_1), (m_2, q_2) \cdots, (m_{t-1}, q_{t-1})\}$, that is the history up to period t consists of the sequence of menus offered until period t, and the sequence of output levels produced by the agent up to period t. We will denote by H_{t-1} the set of histories of possible histories until period $t-1$.

The strategy of the principal in this infinite-horizon adverse selection model is a sequence $\{\sigma_t^P\}_{t=1}^{\infty}$ such that

$$\sigma_t^P : H_{t-1} \to \mathbb{R}_+^L \times \mathbb{R}_+^L$$

where H_{t-1} is the set of histories of the game until period $t-1$. That is, h_t the history up to period t, consists of the sequence of menus offered until period t and the sequence

[5]See for example equation 3.145, page 138 of [10].

of output levels produced by the agent up to period t. The menu of choices offered by the principal in period t generally depends on the past history. Thus given a history h_{t-1} up to time period t, the principal chooses a menu $\sigma_t^P(h_{t-1}) = (P_t(.), q_t(.))$. A strategy of the principal will be denoted by $\sigma^P = \{\sigma_t^P\}_{t=1}^{\infty}$. The agent's strategy is to choose output level in any period t that in general also depends on the past history, as well as on the type θ of the agent. Therefore, the strategy of the agent is a sequence $\{\sigma_t^{A\theta}\}_{t=1}^{\infty}$ such that

$$\sigma_t^{A\theta} : H_{t-1} \times [\underline{\theta}, \bar{\theta}] \to \mathbb{R}_+^L.$$

Given the strategy combination $\sigma = (\sigma^P, \sigma^{A\theta})$ and the history h_{t-1} up to period t, the principal's updated belief will be denoted by the conditional density function $f(\theta|(\sigma, h_{t-1}))$.

At the *beginning of any period* t, the principal offers a menu of contracts m_t. The principal's expected payoff from a menu m_t offered in period t is then given by[6].

$$\int_{\underline{\theta}}^{\bar{\theta}} (S(q_t(\theta)) - P_t(\theta)) f(\theta|(\sigma, h_{t-1})) d\theta.$$

In any period t, the stage game (which is the single-period game in period t), is as follows. First, the principal updates beliefs about the type of the agent given the past history.[7] The principal then offers a menu of contracts that stipulates a monetary transfer P_t for an output level q_t. The agent then, either chooses to accept one of the items in the menu of contracts, and produces the output consistent with the item in the menu of contracts, or refuses the contract. The contract is then executed. That is, the monetary transfer is made.

Over *the infinite horizon* the expected payoff of the principal is the expected discounted sum of the single-period payoffs from the sequence of offers of the principal and those chosen by the agent, and is given by

$$\sum_{t=1}^{\infty} \delta^{t-1} \int_{\underline{\theta}}^{\bar{\theta}} (S(q_t(\theta)) - P_t(\theta)) f(\theta|(\sigma, h_{t-1})) d\theta,$$

where δ is the discount rate of the principal. Therefore, the expected payoff of the principal when the principal's strategy is σ^P and the agent's strategy is $\sigma^{A\theta}$ is

$$U_P^{\infty}(\sigma^P, \sigma^{A\theta}) = \sum_{t=1}^{\infty} \delta^{t-1} [\int_{\underline{\theta}}^{\bar{\theta}} (S(q_t(\theta)) - P_t(\theta)) f(\theta|(\sigma, h_{t-1})) d\theta.$$

[6]It is important to note that the expected payoff of the principal from a menu m_t offered in period t, is dependant on the past history which determines the updated beliefs of the principal.

[7]The agent's type, which the agent learns at the beginning of period 1, stays the same throughout, so the agent knows his type at the beginning of period t.

Similarly, the payoff of the agent over the infinite horizon is the discounted sum of the single-period payoffs. Thus the payoff of agent of type θ over the infinite horizon is

$$\sum_{t=1}^{\infty}\delta^{t-1}U_t(\theta)=\sum_{t=1}^{\infty}\delta^{t-1}(P_t(\theta)-\theta q_t(\theta)).$$

where $(P_t(\theta), q_t(\theta))$ is the contract chosen by the type θ agent in period t. Thus the payoff of the agent when the strategy of the principal and the agent is $(\sigma^P, \sigma^{A\theta})$, is given by

$$U^{\infty}_{A\theta}(\sigma^P,\sigma^{A\theta})=\sum_{t=1}^{\infty}\delta^{t-1}U_t(\theta)(\sigma^P_t(h_{t-1}),\sigma^{A\theta}_t(h_{t-1}))$$

where δ is the discount rate of the agent.

In the infinite-horizon adverse selection model the payoff of the agent is not known to the principal. Hence, the equilibrium concept that we use here is that of a Perfect Bayesian equilibrium[8]. A **Perfect Bayesian equilibrium** is a strategy combination that continues to be an optimal strategy for every player given any history and the updated beliefs of the players, when the beliefs are updated using Bayes' rule.

We note that any strategy combination $(\sigma^P, \sigma^{A\theta})$ generates histories h_t and thus generates a probability over the set of possible histories H_t up to time period t. Thus given a strategy combination, after observing a history h_t the principal is able to update beliefs about the type of the agent using Bayes rule. As already noted these updated beliefs are given by the density function $f(\theta|(\sigma, h_{t-1}))$.

Definition 1 *A strategy combination $\sigma^\star = (\sigma^{P\star}, \sigma^{A\theta\star})$, together with updated beliefs is a* **Perfect Bayesian equilibrium** *of the infinite horizon adverse selection model if*
(i) If beliefs are given by the density function $f(\theta|(\sigma^\star, h_{t-1}))$so that beliefs are updated using the Bayes' rule, and
(ii) for every time period t and for every history h_t up to time period t, the expected payoff of the principal satisfies

$$\int_{\underline{\theta}}^{\bar{\theta}}U^{\infty}_{P}(\sigma^{P\star},\sigma^{A\theta\star}|_{h_t})f(\theta|(\sigma^\star,h_{t-1}))d\theta\geq\int_{\underline{\theta}}^{\bar{\theta}}U^{\infty}_{P}(\sigma^{P},\sigma^{A\theta\star}|_{h_t})f(\theta|(\sigma^\star,h_{t-1}))d\theta$$

for every $\sigma^P|_{h_t}$ and the expected payoff of the agent of each type θ satisfies

$$U^{\infty}_{A\theta}((\sigma^{P\star},\sigma^{A\theta\star})|_{h_t})\geq U^{\infty}_{A\theta}((\sigma^{P\star},\sigma^{A\theta})|_{h_t})$$

for every $\sigma^{A\theta}|_{h_t}$.

[8]Note that as this is a game with incomplete information it can also be viewed as a game with imperfect information, in which a chance move by nature who selects the type of the agent at the beginning of the game cannot be perfectly observed by all the players.

The main result given in Theorem 1 shows the principal offering the static optimal contract in each period is a perfect-Bayesian equilibrium, even though there is no commitment across periods[9]. This equilibrium is a separating equilibrium as the static optimal contract separates types and the principal is able to infer the type of the agent from the choice of the agent. In this equilibrium if the second-best optimal contract fully separates types (that is, there is no bunching of types), then the private information about the agent's type is revealed after the first period. The basic argument that establishes the result shows that a carrot-and-stick strategy works by putting in place penalties when there are deviations, and rewards when contracts are adhered to. In the phases when the payoff of the principal or the agent needs to be shaved because of past deviations, the deviator's payoff is shaved by taking away a small percentage of the information rent or the profit as the case may be.

Theorem 1 *There is a $\bar{\delta} > 0$ such that for all $\delta \geq \bar{\delta} > 0$, the sequence of contracts in which the single-period second-best optimal contract is offered in period 1, and from period 2 onwards, the contract offered is $(\hat{P}(\theta), \hat{q}(\theta))$ if $\hat{q}(\theta)$[10] is the output produced in period 1, is a perfect Bayesian equilibrium. This equilibrium maximizes the expected payoff of the principal.*

Proof: We first note that from (1), the information rent $U(\theta) > 0$ for agents of type $\theta < \bar{\theta}$. Therefore, for an ϵ such that $0 < \epsilon < 1$, we have

$$(1 - \epsilon)U(\theta) > 0 \tag{2}$$

for all $\theta < \bar{\theta}$. Similarly, as the profit of the principal $S(\hat{q}(\theta)) - \hat{P}(\theta)) > 0$ for all type θ, we have

$$(1 - \epsilon)(S(\hat{q}(\theta)) - \hat{P}(\theta))) > 0 \tag{3}$$

for all $\theta \in [\underline{\theta}, \bar{\theta}]$.[11]

We now show that the strategy combination $\{(\hat{\sigma}^P, \hat{\sigma}^{A\theta})\}_{\theta\in[\underline{\theta},\bar{\theta}]}$ described below is a Perfect Bayesian equilibrium.

(i) In period 1 the principal's offer $\hat{\sigma}_1^P$ is the menu of second-best contracts $\{\hat{P}(\theta), \hat{q}(\theta))\}$ for $\theta \in [\underline{\theta}, \bar{\theta}]$.

[9]It is well known that with full commitment across periods the optimal contract that the principal can offer is the second-best optimal contract.

[10]Note that $\hat{q}(\theta)$ is the output in the menu of contracts offered in period 1. This is the output for type θ in the second-best contract.

[11]It is possible that there is a $\theta^\star > \underline{\theta}$ such that for all $\theta \leq \theta^\star$, $(S(\hat{q}(\theta)) - \hat{P}(\theta))) = 0$ and $(S(\hat{q}(\theta)) - \hat{P}(\theta))) > 0$ only for $\theta > \theta^\star$. In that case, the contracts are only offered to types $\theta \geq \theta^\star$.

(ii) From period 2 onwards the principal offers $(\hat{P}(\tilde{\theta}), \hat{q}(\tilde{\theta}))$ if the past history is $(\hat{P}(\tilde{\theta}), \hat{q}(\tilde{\theta}))$ in every period up to $t-1$ then again offer $(\hat{P}(\tilde{\theta}), \hat{q}(\tilde{\theta}))$ in period t.
(iii) If the principal offers $(P, q) \neq (\hat{P}(\tilde{\theta}), \hat{q}(\tilde{\theta}))$ in any period $t \geq 2$ and both the principal and the agent had offered and produced $(\hat{P}(\tilde{\theta}), \hat{q}(\tilde{\theta}))$ in all previous periods, then the agent produces $q = 0$ from time $t+1$ onwards for K periods. This is a punishment phase for the principal.
(iv) If the agent produces $q \neq \hat{q}(\tilde{\theta})$ in any period t and both the principal and the agent had offered and produced $(\hat{P}(\tilde{\theta}), \hat{q}(\tilde{\theta}))$ in all previous periods, then the principal offers the contract $(\hat{q}(\tilde{\theta}), P = \tilde{\theta}\hat{q}(\tilde{\theta}))$,[12] for K-periods. This is a punishment phase for the agent.
(v) If there are no deviations during a punishment phase by either the principal or the agent, and the agent had been the deviator, then after the length of time K, the principal's offer to the agent is

$$(1-\epsilon)U(\tilde{\theta})$$

and the principal gets

$$(S(\hat{q}(\tilde{\theta})) - \hat{P}(\tilde{\theta})) + \epsilon U(\tilde{\theta}).$$

If the principal had been the deviator, then the principal gets

$$(1-\epsilon)(S(\hat{q}(\tilde{\theta})) - \hat{P}(\tilde{\theta})).$$

(vi) *Responses to deviations during a punishment phase:* If the agent does not complete the punishment phase for the principal, then the offer switches to $(\hat{q}(\tilde{\theta}), P = \tilde{\theta}\hat{q}(\tilde{\theta}))$ for a length of time K.[13] After the K-periods the agent is offered $(1-\epsilon)U(\tilde{\theta})$ in each period. This is the *response to a deviation from a punishment phase for the principal*.

If the principal deviates during a punishment phase for the agent, then the agent carries out the threat of producing $q = 0$ for K-periods and then asks for and receives the contract $(\hat{q}(\tilde{\theta}), (1-\epsilon)(S(\hat{q}(\tilde{\theta})) - \hat{P}(\tilde{\theta})))$ in every period after that. And if the principal deviates from that, to again carry out this strategy.
(vii) After a response by the principal to a deviation by the agent from a punishment phase for principal, the offer to the agent becomes

$$(1-\epsilon)U(\tilde{\theta}).$$

(viii) If the principal deviates after a response to a punishment phase, then the punishment phase for the principal starts, after which the offer becomes

$$(1-\epsilon)(S(\hat{q}(\tilde{\theta})) - \hat{P}(\tilde{\theta})).$$

[12] That is, the principal offers a contract that leaves zero information rent to the agent.
[13] The rationale for this is that if the agent does not carry out the full punishment phase, then the principal is encouraged to offer a contract that extracts a lot of surplus from the agent.

(ix) Finally, if the agent deviates after a response to a punishment phase, then the punishment phase for the agent starts again, after which the offer becomes

$$(1-\epsilon)U(\tilde{\theta}).$$

We now proceed to show that the strategy profile $\sigma^\star$ is an equilibrium irrespective of the type of the agent. We first note that as the agent receives $P = \hat{q}(\tilde{\theta})$ for the output level $\hat{q}(\tilde{\theta})$ after a deviation for a number of periods, and then $(1-\epsilon)U(\tilde{\theta})$ after that, the agent can never gain from a deviation.

The principal, however, can make a maximum gain of M[14] in any period by offering a contract that extracts all the information rent from the agent. The strategy of the agent in case the principal deviates, is to produce $q = 0$ for K periods. The principal will not gain from a deviation if

$$\sum_{\nu=1}^{\infty} \delta^{\nu-1}[S(\hat{q}(\tilde{\theta}) - \hat{P}(\tilde{\theta})] \geq M + \delta^K \sum_{\nu=1}^{\infty} \delta^{\nu-1}[S(\hat{q}(\tilde{\theta}) - \hat{P}(\tilde{\theta})](1-\epsilon).$$

That is, if

$$\frac{1-\delta^K}{1-\delta}[S(\hat{q}(\tilde{\theta}) - \hat{P}(\tilde{\theta})] - \frac{\delta^K}{1-\delta}\epsilon \geq M. \tag{4}$$

We note that as $\delta \to 1$, $\frac{1-\delta^K}{1-\delta}$ goes to K and $\frac{\delta^K}{1-\delta}$ goes to ∞. Thus, if δ is chosen to be sufficiently large (say greater than δ_{P1}) the principal cannot gain from deviating.

<u>Deviations from a punishment phase</u>: We now consider deviations from a punishment phase. It should be clear from the analysis of the punishment phase for the agent, an agent cannot gain while the agent is being punished during a punishment phase, irrespective of type as the most the agent can do is refuse to accept the contracts offered. But that leads to a loss of payoff for the agent.

Now consider a deviation made by the agent during a punishment phase when the principal is considered the deviator. Let L_A be the maximum loss every period that the agent sustains.[15] Then the agent's payoff after deviating when $K - t$ $(1 \leq t \leq K+1)$ periods of the punishment phase is left, is equal to

$$0 + \delta^K \sum_{\nu=1}^{\infty} \delta^{\nu-1}(1-\epsilon)U(\tilde{\theta}),$$

[14] The gain of the principal depends on the type of the agent so M should be considered the maximum gain for all possible types of the agent.

[15] Again the loss of the agent will depend on the type of the agent so L_A is the maximum loss for all possible types.

as the agent receives $P = \tilde{\theta}\hat{q}(\tilde{\theta})$ for the output level $\hat{q}(\tilde{\theta})$ for K periods, and then $(1-\epsilon)U(\tilde{\theta})$ in each period after that (see (vi)). If the agent does not deviate, the payoff in the subsequent periods is

$$\delta^{K-t}\sum_{\ell=1}^{\infty}\delta^{\nu-1}[U(\tilde{\theta})+\epsilon(S(\hat{q}(\tilde{\theta}))-\hat{P}(\tilde{\theta}))]-\sum_{\nu=1}^{K-t}\delta^{\nu-1}L_A.$$

Therefore, the agent will not gain by deviating during a punishment phase for the principal, for any t such that $1 \leq t \leq K+1$, if

$$\delta^{K}\sum_{\ell=1}^{\infty}\delta^{\nu-1}[U(\tilde{\theta})+\epsilon(S(\hat{q}(\tilde{\theta})-\hat{P}(\tilde{\theta}))]-\sum_{\nu=1}^{K+1}\delta^{\nu-1}L_A \geq \delta^{K}\sum_{\nu=1}^{\infty}\delta^{\nu-1}(1-\epsilon)U(\tilde{\theta})$$

This reduces to

$$\frac{\delta^K}{1-\delta}\epsilon[U(\tilde{\theta})+S(\hat{q}(\tilde{\theta}))-\hat{P}(\tilde{\theta})] \geq \frac{1-\delta^K}{1-\delta}L_A. \tag{5}$$

We note that

$$U(\tilde{\theta})+S(\hat{q}(\tilde{\theta}))-\hat{P}(\tilde{\theta}) = S(\hat{q}(\tilde{\theta}))-\tilde{\theta}q(\tilde{\theta}).$$

This is the total surplus from production if the agent's type is $\tilde{\theta}$. If we assume that[16]

$$S(\hat{q}(\bar{\theta}))-\bar{\theta}q(\bar{\theta}) > 0,$$

then $S(\hat{q}(\tilde{\theta}))-\tilde{\theta}q(\tilde{\theta}) \geq S(\hat{q}(\bar{\theta}))-\bar{\theta}q(\bar{\theta}) > 0$ for all $\tilde{\theta} \in [\underline{\theta},\bar{\theta}]$. Therefore, (5) can be rewritten as

$$\frac{\delta^K}{1-\delta}\epsilon[S(\hat{q}(\tilde{\theta})-\tilde{\theta}\hat{q}(\tilde{\theta})]) \geq \frac{1-\delta^K}{1-\delta}L_A. \tag{6}$$

In equation (6) as $\delta \to 1$, the expression

$$\frac{1-\delta^K}{1-\delta}$$

goes to K and the expression $\frac{\delta^K}{1-\delta}$ goes to ∞. Hence, there is a $\delta_{A2} : 0 < \delta_{A2} < 1$ such that (6) holds for all $\delta > \delta_{A2}$ and for all $\tilde{\theta} \in [\underline{\theta},\bar{\theta}]$. Again choose $K = K_2$ such that equation (6) holds[17].

Next consider the possibility of a deviation by the principal from a punishment phase for the agent. The principal offers the contract $(\hat{q}(\tilde{\theta}), P = \tilde{\theta}\hat{q}(\tilde{\theta}))$ for K periods thus getting the maximum surplus in each of the K-periods. The principal's payoff from

[16] This therefore assumes that the agent with the highest cost produces a positive surplus.

[17] As we have observed the agent enters this calculation through L_A but these are set so that (6) holds for of all types of the agent, so if K is sufficiently large (6) will hold for agents of all types.

carrying out the full punishment phase for the agent is greater than the payoff from a deviation if,

$$\delta^{K-t}\sum_{\nu=1}^{\infty}\delta^{\nu-1}[S(\hat{q}(\tilde{\theta}))-\hat{P}(\tilde{\theta})+\epsilon U(\tilde{\theta})]\geq M+\delta^{K}\sum_{\nu=1}^{\infty}\delta^{\nu-1}(1-\epsilon)[S(\hat{q}(\tilde{\theta}))-\hat{P}(\tilde{\theta})].$$

That is, if

$$\delta^{K}\sum_{\nu=1}^{\infty}\delta^{\nu-1}[S(\hat{q}(\tilde{\theta}))-\hat{P}(\tilde{\theta})+\epsilon U(\tilde{\theta})]\geq M+\delta^{K}\sum_{\nu=1}^{\infty}\delta^{\nu-1}(1-\epsilon)[S(\hat{q}(\tilde{\theta}))-\hat{P}(\tilde{\theta})].$$

This reduces to

$$\frac{\delta^K}{1-\delta}\epsilon[S(\hat{q}(\tilde{\theta}))-\hat{P}(\tilde{\theta})+U(\tilde{\theta}]\geq M.$$

Hence, we have

$$\frac{\delta^K}{1-\delta}\epsilon[S(\hat{q}(\tilde{\theta}))-\tilde{\theta}\hat{q}(\tilde{\theta})]\geq M. \tag{7}$$

As $\delta\rightarrow 1$ the left hand side of the inequality in (7) goes to ∞. Hence, there is a $\delta_{P2}: 0<\delta_{P2}<1$ such that for all $\delta>\delta_{P2}$ the inequality in (7) holds and the principal cannot gain by deviating during a punishment phase for the agent.

Response to Deviations from a punishment phase: Now consider deviations during a response to a punishment phase. If the agent deviates from a response to a punishment phase for the principal, then the agent cannot gain if for any t such that $1\leq t\leq K$, we have

$$\delta^{K-t}\sum_{\nu=1}^{\infty}\delta^{\nu-1}(U(\tilde{\theta})+(1-\epsilon)[S(\hat{q}(\tilde{\theta}))-\hat{P}(\tilde{\theta})])-\sum_{\nu=1}^{K-t}L_A\geq\delta^{K}\sum_{\nu=1}^{\infty}\delta^{\nu-1}(1-\epsilon)U(\tilde{\theta}).$$

As in the case of deviations from a punishment phase, this will reduce to the inequality in (6). Thus the agent cannot gain for $\delta>\delta_{A2}$.

Similarly, for the principal, a deviation from a response to a punishment phase, while punishing the agent is not profitable if

$$\delta^{K-t}\sum_{\nu=1}^{\infty}\delta^{\nu-1}[S(\hat{q}(\tilde{\theta}))-\hat{P}(\tilde{\theta})+\epsilon U(\tilde{\theta})]\geq M+\delta^{K-t}\sum_{\nu=1}^{\infty}\delta^{\nu-1}(1-\epsilon)[S(\hat{q}(\tilde{\theta}))-\hat{P}(\tilde{\theta})]$$

which reduces to the inequality in (7).

Deviations from post-punishment phases: Finally, consider deviations when the principal is receiving $[\hat{S}(q(\tilde{\theta}))-\hat{P}(\tilde{\theta})](1-\epsilon)$ after a punishment phase, or the agent is receiving $(1-\epsilon)U(\tilde{\theta})$ after a punishment phase.

The principal cannot gain from deviating from $[S(\hat{q}(\tilde{\theta}))-\hat{P}(\tilde{\theta})](1-\epsilon)]$ if

$$\sum_{\nu=1}^{\infty}\delta^{\nu-1}[S(\hat{q}(\tilde{\theta}))-\hat{P}(\tilde{\theta})](1-\epsilon)\geq M+\delta^K\sum_{\nu=1}^{\infty}\delta^{\nu-1}[S(\hat{q}(\tilde{\theta}))-\hat{P}(\tilde{\theta})](1-\epsilon).$$

Noting that $M\leq S(q(\tilde{\theta}))$ which occurs if the principal pays nothing to the agent, we get

$$\frac{1-\delta^K}{1-\delta}[S(\hat{q}(\tilde{\theta}))-\hat{P}(\tilde{\theta})](1-\epsilon))\geq\hat{S}(q(\tilde{\theta}))$$

which gives

$$\frac{1-\delta^K}{1-\delta}(1-\epsilon)\geq\frac{S(\hat{q}(\tilde{\theta}))}{S(\hat{q}(\tilde{\theta}))-\hat{P}(\tilde{\theta})}. \tag{8}$$

Now consider $\eta>0$ that satisfies $\eta\geq\frac{S(\hat{q}(\tilde{\theta}))}{S(\hat{q}(\tilde{\theta}))-\hat{P}(\tilde{\theta})}$ for all $\tilde{\theta}\in[\underline{\theta},\bar{\theta}]$. Let K be sufficiently large and ϵ be sufficiently small so that

$$K(1-\epsilon)>\eta.$$

Then as $\frac{1-\delta^K}{1-\delta}\to K$ as $\delta\to 1$, there is a δ_{P3} such that for all $\delta\geq\delta_{P3}$ the inequality in (8) will hold.

Lastly, consider a deviation of the agent when the agent is receiving $(1-\epsilon)U(\tilde{\theta})$. In this case if the agent deviates the agent gets

$$\delta^K\sum_{\nu=1}^{\infty}(1-\epsilon)U(\tilde{\theta})$$

and if the agent does not deviate, then the agent gets

$$\sum_{\nu=1}^{\infty}(1-\epsilon)U(\tilde{\theta}).$$

Clearly, the agent cannot gain from deviating.

This concludes the part of the proof that shows that neither the principal nor the agent can gain by deviating at any stage. It now remains to observe that the strategies are perfect Bayesian equilibrium strategies. As the optimal single-period contract is such that it completely separates types, therefore, $\hat{q}(\tilde{\theta})\neq\hat{q}(\theta)$ if $\tilde{\theta}\neq\theta$ for all $\tilde{\theta},\theta\in[\underline{\theta},\bar{\theta}]$. In this case consider the updated belief system given by the conditional distribution function[18]

$$F(\theta\,|\,\tilde{\theta})=\begin{cases}0, & \text{for }\theta<\tilde{\theta}\\ 1 & \text{if }\theta\geq\tilde{\theta}\end{cases}$$

[18]This is the distribution function when the updated belief system is given by the *Dirac Measure* $\delta_{\tilde{\theta}}$.

when the contract chosen in period 1 is $(\hat{P}(\tilde{\theta}), \hat{q}(\tilde{\theta}))$ from the menu of the static optimal contracts offered in period 1. This is an updated belief system that is consistent with $\hat{\sigma}$. From the construction of $\hat{\sigma}$ it should be clear that if the principal updates beliefs in this manner, then neither the principal nor the agent, if he is type $\tilde{\theta}$, can gain by deviating from $\hat{\sigma}$ after any history h_t. Further as the updated belief systems[19] in each case is consistent with the equilibrium outcomes, therefore the strategy $\hat{\sigma}$ is a perfect Bayesian equilibrium of the infinite horizon model. ■

2.1 The Optimal Equilibrium Contract with Discrete types

In Theorem 1 we showed that the static optimal contract (often called the *second-best optimal contract*) is offered in period 1 but the model was for a continuum of types. The result also holds when types are discrete as the main arguments do not rely on whether the type of the agent is drawn from a continuum or a discrete set. The main threads of the arguments rely on whether the carrot-and-stick strategy can be applied. In Theorem 1 this depended on the fact that the information rent of the agent was nonincreasing or decreasing in the cost parameter. We observe here that this is true of the discrete type case also and so the result of Theorem 1 also holds when types are discrete.

We recall that in the case when there are $\ell = 1, \cdots, L$ types, with $\theta_1 < \cdots, < \theta_L$, the second-best optimal contract is given by the menu of contracts that maximizes the expected profit of the principal subject to the incentive and participation constraints.

$$\text{Maximize}_{\{q_\ell, P_\ell\}} \quad \sum_{\ell=1}^{L} \nu_\ell (S(q_\ell) - P_\ell))$$

satisfying the following constraints:

$$(i)\ P_\ell - \theta_\ell q_\ell \geq P_{\ell'} - \theta_\ell q_{\ell'} \text{ for all } \ell' \neq \ell \text{ and}$$
$$(ii)\ P_\ell - \theta_\ell q_\ell \geq 0 \text{ for all } \ell.$$

The first set of constraints (i) are the incentive compatibility constraints. The second set of constraints (ii) are participation constraints. We will denote this by the menu $m^{SB} = \{P_\ell^{SB}, q_\ell^{SB}\}_{\ell=1}^{L}$.

Corollary 1 Discrete Types *There is a $\bar{\delta} > 0$ such that for all $\delta \geq \bar{\delta} > 0$, the sequence of contracts in which the single-period second-best optimal contract is offered in period 1, and from period 2 onwards, the contract offered is $(\hat{P}, \hat{q})$ if $\hat{q}$[20] is the output produced*

[19]As before, we do not specify the updated beliefs when $\theta \neq \tilde{\theta}$ for $\theta \in [\underline{\theta}, \bar{\theta}]$ as this occurs with probability zero in equilibrium.

[20]Note that $\hat{q} = q_\ell^{SB}$ for some ℓ and is one of the outputs in the menu of contracts offered in period 1.

in period 1, is a perfect Bayesian equilibrium. This equilibrium maximizes the expected payoff of the principal.

Proof: We observe that for

$$P_\ell - \theta_\ell q_\ell > 0, \text{ for all } \ell = 1, \cdots, L-1. \tag{9}$$

Here we assume that $q_L > 0$. Since each contract in the menu satisfies the participation constraints, we have

$$P_L - \theta_L q_L \geq 0.$$

Then because of incentive compatibility, for all $\ell = 1, \cdots, L-1$, we have

$$P_\ell - \theta_\ell q_\ell \geq P_L - \theta_\ell q_L > P_L - \theta_L q_L \geq 0$$

so that

$$P_\ell - \theta_\ell q_\ell > 0. \tag{10}$$

We next observe that

$$p_\ell - \theta_\ell q_\ell \geq P_{\ell+1} - \theta_\ell q_{\ell+1} > P_{\ell+1} - \theta_{\ell+1} q_{\ell+1} \tag{11}$$

from the incentive compatibility constraints and the fact that $\theta_\ell < \theta_{\ell+1}$.

The result now follows from noting that a strategy combination similar to $\sigma^\star$ of Theorem 1 that relies on (10) and (11) is a perfect Bayesian equilibrium. ∎

In the next example we illustrate the perfect Bayesian equilibrium in the case when the agent is of two types; a low-cost type with $\theta = \underline{\theta}$ and high-cost type with $\theta = \bar{\theta}$.

Example 1 *The two-type case.*

It is well known in this case that the second-best contracts are given by the menu of contracts shown below.
(i) For type $\bar{\theta}$, the output is $\hat{q}(\bar{\theta})$ and the payment is $\hat{P}(\bar{\theta}) = \bar{\theta}\hat{q}(\bar{\theta})$.
(ii) For the low-cost type, the output is $\hat{q}(\underline{\theta}) = q^\star(\underline{\theta})$. The payment is $\hat{P}(\underline{\theta}) = \underline{\theta} q^\star(\underline{\theta}) + (\bar{\theta} - \underline{\theta})\hat{q}(\bar{\theta})$.

Here $q^\star(\underline{\theta})$ is the first-best output that maximizes the principal's profit if the type is $\underline{\theta}$. That is, $q^\star(\underline{\theta}) \in \text{argmax}(\text{S(q)} - \underline{\theta}\text{q})$. The amount $(\bar{\theta} - \underline{\theta})\hat{q}(\bar{\theta})$ is the information rent that has to be paid to the low-cost type in order to satisfy the incentive constraints. The output $\hat{q}(\bar{\theta}) < q^\star(\bar{\theta})$ where $q^\star(\bar{\theta})$ is the output that maximizes the principal's profit if the type is $\bar{\theta}$.

The Perfect Bayesian equilibrium: The perfect Bayesian equilibrium strategy for this case is as follows.
(i) The principal offers the menu of the two contracts in period 1. The low-cost type chooses the contract meant for the low-cost type and produces $q^\star(\underline{\theta})$ and receives the payment $\underline{\theta}q^\star(\underline{\theta}) + (\bar{\theta} - \underline{\theta})\hat{q}(\bar{\theta})$. The high-cost agent chooses the contract meant for the high-cost type and produces the output $\hat{q}(\bar{\theta})$ and receives the payment $\bar{\theta}\hat{q}(\bar{\theta})$.
(ii) Observing the output level produce in period 1, the principal updates beliefs about the type of the agent. If the output was $\hat{q}(\underline{\theta})$ then the updated belief is that the agent is of type $\underline{\theta}$ with probability 1. If the output is $\hat{q}(\bar{\theta})$ then the agent is paid $\bar{\theta}\hat{q}(\bar{\theta})$ and the updated belief is that the agent is of type $\bar{\theta}$ with probability 1.
(iii) In the next period, the principal offers one of the two contracts depending on the updated belief, and the agent accepts the contract. If this happens then the equilibrium strategy specifies that this is the contract offered and accepted every period.
(iv) The principal deviates in any period if the principal offers a different contract. For example the principal may want to offer a contract like $P = \underline{\theta}q^\star(\underline{\theta})$ for the output $q^\star(\underline{\theta})$ after updating beliefs to one in which the principal believes that the agent is of type $\underline{\theta}$. This then leads to the punishment phase for the principal in which the agent produces $q = 0$ for K-periods and then only accepts a contract in which the payoff of the principal is lowered by a factor of $(1 - \epsilon)$.
(v) A deviation by the agent is to reject the contract offered. In this case, the principal offers a contract in which the principal receives the entire surplus from the contract for K-periods and then offers a contract in which the payoff of the agent is lowered by a factor of $(1 - \epsilon)$.

It is shown in the proof of Theorem 1 that neither the principal nor the agent can gain from deviations when such punishment strategies are used.

However, in a perfect Bayesian equilibrium such punishment strategies must be credible. That is, deviating from such punishment strategies should also not be profitable. This is quite clear for the punishment strategy for the principal. For the agent the motivation for carrying out the punishment strategy is the possibility of not only the payment from the second-best optimal contract, but in addition a fraction ϵ of the surplus of the principal, after K-periods. Thus carrying out the punishment strategy by the agent is worthwhile, if the agent is sufficiently patient, (the discount rate δ is sufficiently high). This is further guaranteed by the fact that if the agent fails to fully carry out the punishment strategy, then the principal will offer terms that take away much of the information rent of the agent. The principal will first offer contracts in which the agent receives no information rent for K-periods, and then is given a payment that lowers the information rent by a factor of $(1 - \epsilon)$. Thus, there is a very strong incentive for the agent to carry out the punishment strategy. This, therefore, makes the threat of the punishment strategy credible.

Finally, we note something of interest about the type $\bar{\theta}$. For this type, there is no information rent and thus no incentive for the principal to extract any of the information rent. The principal gets the entire surplus and thus would be content to offer the second-best optimal contract to this type in each period without deviating from it. ■

3 Applications

While adverse selection issues arise in a wide variety of cases, we look at two applications that are of special interest. The first application looks at the model of regulating a firm that is discussed in Laffont and Tirole [11]; this model has both the elements of moral hazard and adverse selection and therefore is not a standard model of adverse selection. The second application examines the case of a firm selling to a consumer whose valuation for the good is not precisely known by the firm. This example is of great interest in industrial organization and fits the framework of the standard adverse selection model.

3.1 Regulation of a publicly owned firm

Laffont and Tirole [11] examined a two-period model in which the regulator of a publicly held firm pays an amount s to compensate the manager for his effort. The model is one in which the regulator observes the cost c of the firm, where the cost is given by

$$c = \beta - e,$$

where β is a cost parameter known only to the manager of the firm, and e is the effort of the manager. While the regulator observes the cost c, the regulator does not observe the effort level e of the manager, and only knows that $\beta \in [\underline{\beta}, \bar{\beta}]$ and that the distribution function of β is $F(.)$ with density function $f(.)$.

The welfare of the consumer is given by

$$u - (1 + \lambda)(s + c)$$

where u is the utility of the consumer, s is the payment made to the manager of the firm, and λ is a measure of the distortion that results from the fact that the regulator has to public funds to pay the manager. The utility or payoff of the manger of the firm is given by

$$s - \psi(e)$$

where $\psi(e)$ is the disutility of the manager from putting forth the effort level e.

Laffont and Tirole examined the nature of the optimal contracts when there are two periods and the regulator cannot commit to a second-period incentive scheme in the first period. They show in proposition 1 that for *any first period incentive scheme* $s(.)$*, there exists no fully separating continuation equilibrium.*

In analyzing such a model it is useful to look at the benchmark case when the parameter β is known to the regulator. As the regulator would want to maximize the total surplus, the regulator would offer the contract that

$$\max_{s,e}[u - (1+\lambda)(s+\beta-e)) + s - \psi(e)] \tag{12}$$

subject to

$$s \geq \psi(e). \tag{13}$$

Equation (13) gives the participation constraint of the manager and it shows that the manager will not put in any effort unless the manager receives a payment that is at least as much as the disutility from the effort. We note that at the optimal solution

$$s = \psi(e)$$

so that the problem reduces to

$$\max_{e}[u - (1+\lambda)(\psi(e)+\beta-e))]$$

which gives

$$\psi'(e^{\star}) = 1. \tag{14}$$

From this we get the first-best outcome

$$s^{\star} = \psi(e^{\star}). \tag{15}$$

In the case in which the regulator only knows that $\beta \in [\underline{\beta}, \bar{\beta}]$ with the distribution $F(.)$, the contract offered by the regulator is given by a schedule $s(c)$, where the payment s is a function of the observed cost c. For any contract schedule $s(c)$, the manager will choose e and thus c so as to maximize the rent

$$\{s(c) - \psi(\beta - c)\}.$$

The information rent of the manager is then

$$\Pi(\beta) = \max_{c}\{s(c) - \psi(\beta - c)\} = \max_{e}\{s(\beta - e) - \psi(e)\}$$

As observed in Laffont and Tirole [11], $\Pi(\beta)$ is a non-increasing function as the manager of a more efficient firm can always work less at the same cost. That is, given any $s(c)$, if $\beta' > \beta$, then there is $e' < e$ such that $\beta' - e' = c$ so that $\psi(e') < \psi(e)$ and $\Pi(\beta') \leq \Pi(\beta)$.

The objective of the regulator is to maximize the expected total surplus given by

$$W = \int_{\underline{\beta}}^{\bar{\beta}} \{u - (1+\lambda)(s + \beta - e) + s - \psi(e)\} f(\beta) d\beta. \tag{16}$$

Let $s^\star(c)$ denote the optimal incentive scheme. Let $e^\star(\beta)$ denote the optimal effort level of the manager in response to the incentive scheme $s^\star(c)$. Then, the information rent of the manager under the optimal incentive scheme is given by

$$\Pi^\star(\beta) = \max_c \{s^\star(c) - \psi(\beta - c)\} = \max_e \{s^\star(\beta - e) - \psi(e)\}.$$

In Laffont and Tirole [11] it is observed that under the monotone hazard rate assumption that $\frac{F_1}{f_1}$ is nondecreasing, that (i) $e^\star(\beta)$ is nonincreasing and thus fully separates types, (ii) if $\tilde{\beta}$ is the supremum in $[\underline{\beta}, \bar{\beta}]$ of types that would make positive rent from participating then $\Pi^\star(\tilde{\beta}) = 0$.

The next result shows that when there is the possibility of contracts over an indeterminate number of periods, that is over an infinite horizon, then the optimal static contract $s^\star(.)$ is offered in a perfect Bayesian equilibrium.

Proposition 1 *If the firm is regulated over an infinite horizon with no commitment across periods, there is a $\bar{\delta}$ such that for all $\delta \geq \bar{\delta}$, offering the fully separating optimal static contract $s^\star(.)$ in period 1, followed by a continuation of the contract chosen by the manager in period 1, is a perfect Bayesian equilibrium.*

Proof: We first observe that for $\beta \in [\underline{\beta}, \tilde{\beta})$, $\Pi^\star(\beta) > 0$ so that for a small $\epsilon > 0$,

$$(1 - \epsilon)\Pi^\star(\beta) > 0.$$

Next let $c^\star(\beta) = \beta - e^\star(\beta)$. Then, $s^\star(c^\star(\beta))$ denotes the payment for the manager of type β in the static optimal contract. If now the information rent of the manager is reduced by ϵ to $(1-\epsilon)\Pi^\star(\beta)$, then the payoff of the regulator is given by

$$W^\star(\beta) = u - (1+\lambda)((1-\epsilon)s^\star(c^\star(\beta)) + \beta - (1-\epsilon)e^\star(\beta)) + (1-\epsilon)\Pi^\star(\beta).$$

We note that when the rent of the manager is reduced by ϵ, the payoff of the manager is

$$(1-\epsilon)\Pi^\star(\beta) = (1-\epsilon)s^\star(c^\star(\beta)) - (1-\epsilon)e^\star(\beta))$$

so that both the payment $s^\star(c^\star(\beta))$ as well as the effort level $e^\star(\beta)$ is reduced by ϵ. This means that if the manager is paid $(1-\epsilon)\Pi^\star(\beta)$, then the social welfare (the payoff of the regulator) is

$$u - (1+\lambda)((1-\epsilon)s^\star(c^\star(\beta)) + \beta - (1-\epsilon)e^\star(\beta)) + (1-\epsilon)(s^\star(c^\star(\beta) - e^\star(\beta)).$$

This reduces to

$$W^\star(\beta) + \epsilon(s^\star(c^\star(\beta)) - e^\star(\beta)) = W^\star(\beta) + \epsilon\Pi^\star(\beta) > W^\star(\beta). \tag{17}$$

Similarly, when the payment of the manager is increased by an amount ϵ, the social welfare is given by

$$u - (1+\lambda)((1+\epsilon)s^\star(c^\star(\beta)) + \beta - e^\star(\beta)) + (1-\epsilon)(s^\star(c^\star(\beta)) - e^\star(\beta)).$$

Here, the effort level remains the same at $e^\star(\beta)$ but the payment is increased. This reduces to

$$W^\star(\beta) - (1+\lambda)\epsilon s^\star(c^\star(\beta)) + \epsilon s^\star(c^\star(\beta)) = W^\star(\beta) - \lambda\epsilon s^\star(c^\star(\beta)) < W^\star(\beta). \tag{18}$$

We now show that the strategy combination $\{(\hat{\sigma}^R, \hat{\sigma}^M(\beta)\}$ described below is a Perfect Bayesian equilibrium.
(i) In period 1 the regulator's payment schedule $\hat{\sigma}_1^R$ is the static optimal contract $s^\star(c)$.
(ii) From period 2 onwards the regulator offers $s^\star(c^\star(\beta))$ if the past history is $(s^\star(c^\star(\beta)), c^\star(\beta))$ in every period up to $t-1$.
(iii) If the regulator offers $s \neq s^\star(c^\star(\beta))$ in any period $t \geq 2$ and both the payment and realized cost is $(s^\star(c^\star(\beta)), c^\star(\beta))$ in all previous periods, then the manager puts in effort $e = 0$ from time $t+1$ onwards for K periods. This is a phase I punishment strategy.
(iv) If the realized cost is $c \neq c^\star(\beta)$ in any period t and both the payment and realized cost is $(s^\star(c^\star(\beta)), c^\star(\beta))$ in all previous periods in all previous periods, then the principal pays $s = s^\star(c^\star(\underline{\beta}))$ for K periods after that. This is a phase I punishment for the agent.
(v) If there are no deviations during a phase I punishment by either the regulator or the manager, and the manager had been the deviator, then after the length of time K, the regulator's payment is set so that the rent of the manager is

$$(1-\epsilon)\Pi^\star(\beta)$$

and the social welfare (the payoff of the regulator) from (15) is

$$W^\star(\beta) + \epsilon\Pi^\star(\beta).$$

If the regulator had been the deviator, then the rent of the manager is

$$(1+\epsilon)\Pi^{\star}(\beta)$$

and the social welfare (the payoff of the regulator) from (16) is

$$W^{\star}(\beta) - \epsilon\Pi^{\star}(\beta).$$

(vi) If the manager deviates during a phase I punishment for the regulator, then the payment switches to $s = s^{\star}(c^{\star}(\underline{\beta}))$ for K periods. If the regulator deviates while punishing the manager during a phase I punishment, then the effort of the manager is $e = 0$ for the next K periods. Such a punishment is a phase II punishment.

(vii) After a phase II punishment for the regulator, the manager gets

$$(1+\epsilon)\Pi^{\star}(\beta)$$

and the social welfare (the payoff of the regulator) from is

$$W^{\star}(\beta) - \epsilon\Pi^{\star}(\beta).$$

After a phase II punishment for the manager the rent of the manager is

$$(1-\epsilon)\Pi^{\star}(\beta)$$

and the social welfare (the payoff of the regulator) is

$$W^{\star}(\beta) + \epsilon\Pi^{\star}(\beta).$$

(viii) If the regulator deviates after a phase II punishment, then the phase I punishment for the regulator starts after which the manager gets

$$(1+\epsilon)\Pi^{\star}(\beta)$$

and the social welfare (the payoff of the regulator) from is

$$W^{\star}(\beta) - \epsilon\Pi^{\star}(\beta).$$

(ix) Finally, if the manager deviates after a phase II punishment, then the phase I punishment for the manager starts again after the rent of the manager is

$$(1-\epsilon)\Pi^{\star}(\beta)$$

and the social welfare (the payoff of the regulator) is

$$W^{\star}(\beta) + \epsilon\Pi^{\star}(\beta).$$

The arguments that this is a perfect Bayesian equilibrium are now similar to the proof of Theorem 1 which shows that because of the nature of the punishment to deviations, and the response to deviations from punishments, there is no possibility of a profitable deviation.

The updated belief of the regulator is given by prob.$(\beta|c = c^{\star}(\beta)) = 1$ for $\beta \in [\underline{\beta}, \bar{\beta}]$ if the cost realized in period 1 is $c^{\star}(\beta)$. ∎

3.2 Optimal Nonlinear Prices

Here we study the well known optimal nonlinear pricing model in which a seller set prices for buyers with valuations for the good or service that are drawn from some interval $[\underline{\theta}, \bar{\theta}]$. The seller knows that the distribution of the valuation of the buyer is given by a distribution function $F(.)$ with density function $f(.)$ where $f(\theta) > 0$ for $\theta \in [\underline{\theta}, \bar{\theta}]$. The utility of a type θ buyer is given by

$$U(q, T, \theta) = \theta v(q) - T(q),$$

where $\theta v(q)$ is the value to the buyer of type θ from q amount of the good or service, and $T(q)$ is the amount paid for the amount q. The profit of the seller is given by

$$\pi = T(q) - cq,$$

where $T(q)$ is the payment for q units and c is the unit cost of production so that cq is the total cost of producing q units.

The seller's problem can be written as

$$\max_{q(\theta),\, T(q(\theta))} \int_{\underline{\theta}}^{\bar{\theta}} (T(q(\theta)) - cq(\theta))f(\theta)d\theta$$

subject to the participation constraints

$$\theta v(q(\theta)) - T(q(\theta)) \geq 0$$

and

$$\theta v(q(\theta)) - T(q(\theta)) \geq \theta v(q(\tilde{\theta})) - T(q(\tilde{\theta}))$$

for all $(\theta, \theta') \in [\underline{\theta}, \bar{\theta}] \times [\underline{\theta}, \bar{\theta}]$. Let $U(\theta)$ denote the information rent of type θ. Then at the optimal solution (a comprehensive treatment of this is to be found in Bolton and Dewatripont [2]), we have

$$U(\theta) = \int_{\underline{\theta}}^{\theta} v(q(s))ds$$

so that the information rent is increasing in the buyer's valuation. As $U(\underline{\theta}) = 0$, we have

$$U(\theta) = \int_{\underline{\theta}}^{\theta} v(q(s))ds > 0 \tag{19}$$

for $\theta > \underline{\theta}$. Further,

$$[\theta - \frac{1 - F(\theta)}{f(\theta)}]v'(q(\theta)) = c. \tag{20}$$

The next result shows that if the seller and buyer expect to trade over an indefinitely long number of periods with a sufficiently high probability, then offering the static optimal contract in period 1 is a perfect Bayesian equilibrium.

Proposition 2 *If the seller and buyer trade over an infinite horizon with no commitment across periods, there is a $\bar{\delta}$ such that for all $\delta \geq \bar{\delta}$[21], offering the fully separating second-best optimal contract $(\hat{q}(\theta), \hat{T}(\hat{q}(\theta))$ in period 1, followed by a continuation of the contract chosen by the buyer in period 1, is a perfect Bayesian equilibrium of the infinite horizon model.*

Proof: We note that from (19,) the information rent of the buyers with valuations $\theta > \underline{\theta}$ are positive. Further, it should be clear that the profit of the seller for each θ, given by

$$\hat{T}(\hat{q}(\theta)) - c\hat{q}(\theta),$$

are positive. Therefore, from Theorem 1 it follows that a strategy combination like $\sigma^\star$ in which in period 1 the seller offers the price and quantity schedule $(\hat{T}(\hat{q}(\theta)), \hat{q}(\theta))$ in period 1, followed by the offer $(\hat{T}(\hat{q}(\tilde{\theta})), \hat{q}(\tilde{\theta}))$ if the buyer chose $(\hat{T}(\hat{q}(\tilde{\theta})), \hat{q}(\tilde{\theta})))$ from the schedule offered in period 1.

Because of (19) the carrot and stick strategy used in Theorem 1 can now be used to show that $\sigma^\star$ is a perfect Bayesian equilibrium. ■

4 Conclusion

The framework here is one in which explicit contracts are written in each period with no commitment across periods, and the interaction does not have a definite terminal period. The results here should, therefore, be seen as indicating the possible nature of contracts when the time horizon extends indefinitely into the future, and how that

[21] Here it may be useful to interpret the discount factor δ as the probability with which the buyer comes back to the seller. It is also worth mentioning that in this case the condition that there is no commitment across periods is important as the buyer can always choose not to trade in the future.

affects the nature of the contracts. While we have used the traditional interpretation of the discount factor δ as a factor that discounts future payoffs, it is possible the discount factor is the probability with which the principal and the agent will engage in contract negotiations in the future. If that probability is sufficiently high then the carrot and stick kind of strategy can be used effectively.

The result presented here has interesting applications as is evident from the two applications that are discussed here. In both the applications the condition of no commitment across periods is the most reasonable assumption to make. In the case of the publicly held firm, while commitment over a short period may be possible, commitment over long periods do not work. In the example of nonlinear pricing when the seller makes offers to a buyer, it would be hard to prevent a buyer from changing his/her choice over different periods, or prevent the buyer from leaving the market altogether.[22] The result also illustrates some major elements in many labor markets. For instance, the separating contract in which the second-best contract is offered, is possibly what occurs, when a worker is given a menu of contracts during a probationary period, after which, the worker's wage or salary in the subsequent periods is determined on the basis of what occurred during the probationary period.

The idea that the principal infers the type of the agent, and then uses that to offer a contract that includes the information of the rent of the second-best contract, is also indicated by the perks and the guarantee of quality offered to clients who are viewed as high valuation clients.[23] Offers of this kind also may be a method of indicating that revealing a preference for higher quality would be rewarding, and would lead to a continual flow of rents.

It is also worth noting that in the perfect Bayesian equilibrium, the more productive workers receive higher payments due to the information rent. This is a feature that is in common with Espinosa and Rhee [5] that high efficiency wages occur in equilibrium for the more productive agents. Although Espinosa and Rhee [5] address the question of shirking in a repeated game framework, and thus deals with issue of moral hazard, it is intriguing that an argument for a wage premium can be made not only for the case of

[22] Such contracts are quite common in the telecommunications industry in which a buyer can choose from a menu that includes an option in which the buyer has unlimited minutes, but with a higher monthly fee, as well as an option with a fixed number of minutes with a lower monthly fee. The contract is valid for a limited time, and the buyer is free to leave or choose a different contract after that. The seller too can change the terms of the contract in the next period, thus there is no commitment across periods. The buyer can also choose to stay with the seller for many periods.

[23] For example the Hilton Honors program can be viewed from this perspective. Airlines too frequently find ways to reward the consumers who show a preference for higher quality, or more frequent use of the services.

moral hazard, but also in the case of repeated adverse selection.

Finally, it needs mentioning that an interesting feature of the result is how quickly information about the type of the agent is revealed in equilibrium. Clearly, the only way in which the principal can elicit information about the true type of the agent is to pay the information rent. As there is no advantage for the principal to delay the revelation of the information, since the principal benefits from correctly inferring the agent's type, the principal should do it at the earliest opportunity. In fact, the equilibrium shows that the optimal strategy of the principal is to pay the right amount of information rent that elicits the desired response from the agent, and the sooner this is done the more profitable it is for the principal. The agent too does not gain by waiting to disclose the information about his type as the agent is appropriately rewarded for revealing the information.

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